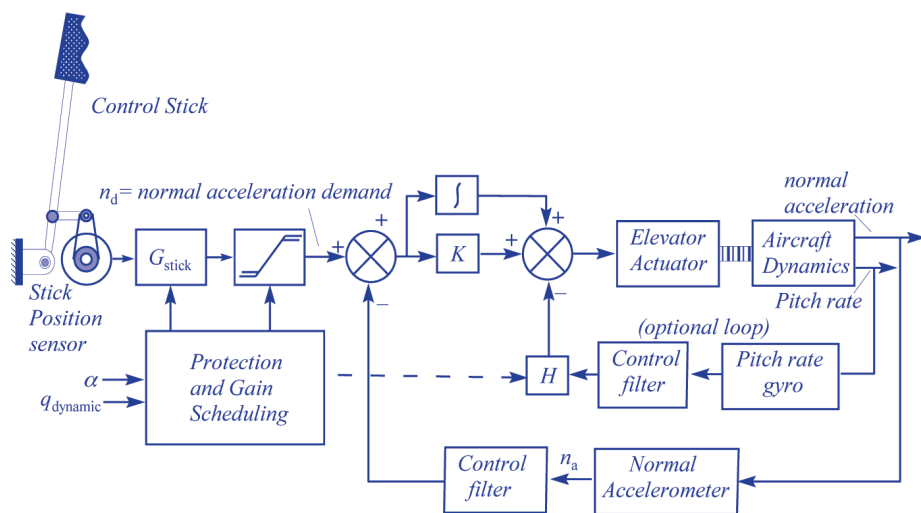




# Flight Dynamics, Simulation, and Control

## For Rigid and Flexible Aircraft





**Flight Dynamics,  
Simulation, and Control**  
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Flexible Aircraft**



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**Ranjan Vepa**



**CRC Press**

Taylor & Francis Group  
Boca Raton London New York

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CRC Press  
Taylor & Francis Group  
6000 Broken Sound Parkway NW, Suite 300  
Boca Raton, FL 33487-2742

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Version Date: 20140707

International Standard Book Number-13: 978-1-4665-7336-9 (eBook - PDF)

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*To my teachers, Hans Wagner, Horst Leipholz,  
Holt Ashley, Art Bryson and Geoff Hancock*





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## *List of Acronyms*

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AC	Aerodynamic centre
ADF	Automatic direction finding
amc	Aerodynamic mean chord
AR	Aspect ratio
BDF	Backward difference formula
CG	Centre of gravity
CH	Cooper–Harper (rating)
CM	Centre of mass
CP	Centre of pressure
DME	Distance measuring equipment
EFIS	Electronic flight information system
EIS	Electronic information system
EPR	Engine pressure ratio
FBW	Fly by wire
FCU	Flight control unit
FDAU	Flight data acquisition unit
FMGS	Flight management and guidance system
GPS	Global positioning system
HSI	Horizontal situation indicator
HUD	Head-up displays
IAS	Indicated airspeed
IFS	In-flight simulation
ILS	Instrument landing system
INS	Inertial navigation system
NDF	Numerical differentiation formula
NP	Neutral point
PD	Proportion derivative
PID	Proportional, integral, derivative
PIO	Pilot-induced oscillation
psfc	Power-specific fuel consumption
RMI	Radio magnetic indicator
SISO	Single input, single output
TCAS	Traffic collision avoidance system
TR	Trapezoidal rule
Tsfc	Thrust-specific fuel consumption
VHF	Very high frequency
VOR	VHF omni-range or vestibulo-ocular reflex

