Mathematical Surveys and Monographs

Volume 9

Linear Approximation

Arthur Sard

American Mathematical Society

http://dx.doi.org/10.1090/surv/009

Other Titles in This Series

45 **George M. Bergman and Adam O. Hausknecht,** Cogroups and co-rings in categories of associative rings, 1996

44 **J. Amorós, M. Burger, K. Corlette, D. Kotschick, and D. Toledo,** Fundamental groups of compact Kähler manifolds, 1996

43 **James E. Humphreys,** Conjugacy classes in semisimple algebraic groups, 1995

42 **Ralph Freese, Jaroslav Ježek, and J. B. Nation,** Free lattices, 1995

41 **Hal L. Smith,** Monotone dynamical systems: an introduction to the theory of competitive and cooperative systems, 1995

40.2 **Daniel Gorenstein, Richard Lyons, and Ronald Solomon,** The classification of the finite simple groups, number 2, 1995

40.1 **Daniel Gorenstein, Richard Lyons, and Ronald Solomon,** The classification of the finite simple groups, number 1, 1994

39 **Sigurdur Helgason,** Geometric analysis on symmetric spaces, 1993

38 **Guy David and Stephen Semmes,** Analysis of and on uniformly rectifiable sets, 1993

37 **Leonard Lewin, Editor,** Structural properties of polylogarithms, 1991

36 **John B. Conway,** The theory of subnormal operators, 1991

35 **Shreeram S. Abhyankar,** Algebraic geometry for scientists and engineers, 1990

34 **Victor Isakov,** Inverse source problems, 1990

33 **Vladimir G. Berkovich,** Spectral theory and analytic geometry over non-Archimedean fields, 1990

32 **Howard Jacobowitz,** An introduction to CR structures, 1990

31 **Paul J. Sally, Jr. and David A. Vogan, Jr., Editors,** Representation theory and harmonic analysis on semisimple Lie groups, 1989

30 **Thomas W. Cusick and Mary E. Flahive,** The Markoff and Lagrange spectra, 1989

29 **Alan L. T. Paterson,** Amenability, 1988

28 **Richard Beals, Percy Deift, and Carlos Tomei,** Direct and inverse scattering on the line, 1988

27 **Nathan J. Fine,** Basic hypergeometric series and applications, 1988

26 **Hari Bercovici,** Operator theory and arithmetic in H^∞, 1988

25 **Jack K. Hale,** Asymptotic behavior of dissipative systems, 1988

24 **Lance W. Small, Editor,** Noetherian rings and their applications, 1987

23 **E. H. Rothe,** Introduction to various aspects of degree theory in Banach spaces, 1986

22 **Michael E. Taylor,** Noncommutative harmonic analysis, 1986

21 **Albert Baernstein, David Drasin, Peter Duren, and Albert Marden, Editors,** The Bieberbach conjecture: Proceedings of the symposium on the occasion of the proof, 1986

20 **Kenneth R. Goodearl,** Partially ordered abelian groups with interpolation, 1986

19 **Gregory V. Chudnovsky,** Contributions to the theory of transcendental numbers, 1984

18 **Frank B. Knight,** Essentials of Brownian motion and diffusion, 1981

17 **Le Baron O. Ferguson,** Approximation by polynomials with integral coefficients, 1980

16 **O. Timothy O'Meara,** Symplectic groups, 1978

15 **J. Diestel and J. J. Uhl, Jr.,** Vector measures, 1977

14 **V. Guillemin and S. Sternberg,** Geometric asymptotics, 1977

13 **C. Pearcy, Editor,** Topics in operator theory, 1974

This page intentionally left blank

MATHEMATICAL
Surveys and Monographs

Volume 9

Linear Approximation

Arthur Sard

American Mathematical Society
Providence, Rhode Island

The writing of this book was supported in part by the Aeronautical Research Laboratories and the Office of Scientific Research, both of the United States Air Force.

Library of Congress Catalog Number: 63-11988
International Standard Book Number 0-8218-1509-1
International Standard Serial Number 0076-5376

---

10 9 8 7 6 5 4 01 00 99 98 97 96

Preface

The form in which I have written "Linear approximation" seems to me to be suited to the subject, and to the use of mathematicians, scientists, and engineers.

Readers interested in the applications may wish to start with the illustrative examples and the statements of theorems. All readers are urged to skip boldly and to sample where they will.

I am grateful to my wife and to many men and women for help and teaching. In the words of David, Psalm 16, 6,

חבלים נפלו לי בנעימים אף נחלת שפרה עלי.

The lines are fallen unto me in pleasant places;
yea, I have a goodly heritage.

ARTHUR SARD

July 21, 1962

QUEENS COLLEGE
THE CITY UNIVERSITY OF NEW YORK
FLUSHING, NEW YORK

Second Preface

There has been much done since 1963. Rather than attempting a synopsis, I will list a number of my papers, in which detailed references are given to work of Atteia, Barnhill, DeBoor and Lynch, Delvos, M. Golomb and Weinberger, Holladay, Schempp, Schoenberg, Walsh, Ahlberg and Nilson, myself, and others.

The "special formulas" of Chapter 2 are now called "natural splines" or "minimizing splines". The open question discussed on page 107 has been resolved affirmatively: The natural splines are the best broad formulas. See references [3] and [9] of the Supplementary Bibliography. Furthermore, the best formulas of Parts 1, 3, and 4 of Chapter 2 are derivable from Part 2, since the best approximation of $\tilde{G}x$ is Gy, where y is the natural spline approximation of x and G is any admissible operator. The direct calculation of Gy may, however, be easier than that of y.

Papers [1] and [7] refer to Chapter 6; paper [6] refers to Chapter 9.

I wish to thank Valerie Meyer for her great help.

ARTHUR SARD

August 15, 1980

BINNINGEN
SWITZERLAND

Table of Contents