AIRCRAFT STRUCTURES

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PREFACE

The subject of aircraft structures includes a wide range of topics. Most of the classical methods of analysis for heavy structures must be considered as well as methods which have particular application to lightweight structures. It is obviously impossible to treat all phases of the analysis and design of aircraft structures in a single book. In this book, which is written as an undergraduate college text, an attempt is made to emphasize basic structural theory which will not change as new materials and new construction methods are developed. Most of the theory is applicable for any design requirements and for any materials. The design engineer may then supplement this theory with the detail design specifications and the material properties which are applicable to his particular airplane.

It is believed that most of the serious mistakes made by college students and by practicing engineers result from errors in applying the simple equations of statics. Heavy emphasis is therefore placed on the application of the elementary principles of mechanics to the analysis of aircraft structures. The problems of deflections and statically indeterminate structures are treated in later chapters. The topics of air-load distribution and flight loading conditions require a knowledge of aerodynamics. These topics are therefore placed in the latter part of the book because students usually study the prerequisite aerodynamics topics in concurrent courses.

The author appreciates the suggestions of Dr. Joseph Marin, Dr. Alexander Klemin, Prof. Raymond Schneyer, and George Cohen, who read and criticized the first part of the manuscript; Dr. P. E. Hemke, Dr. A. A. Brielmaier, and C. E. Duke also made valuable suggestions and criticisms.

DAVID J. PEERY

STATE COLLEGE, PA.
August, 1949

CONTENTS

PREFACE . . . v
1. EQUILIBRIUM OF FORCES . . . 1
2. SPACE STRUCTURES . . . 20
3. INERTIA FORCES AND LOAD FACTORS . . . 46
4. MOMENTS OF INERTIA, MOHR'S CIRCLE . . . 73
5. SHEAR AND BENDING-MOMENT DIAGRAMS . . . 100
6. SHEAR AND BENDING STRESSES IN SYMMETRICAL BEAMS . . . 113
7. BEAMS WITH UNSYMMETRICAL CROSS SECTIONS . . . 153
8. ANALYSIS OF TYPICAL MEMBERS OF SEMIMONOCOQUE STRUCTURES . . . 181
9. SPANWISE AIR-LOAD DISTRIBUTION . . . 213
10. EXTERNAL LOADS ON THE AIRPLANE . . . 250
11. MECHANICAL PROPERTIES OF AIRCRAFT MATERIALS . . . 271
12. JOINTS AND FITTINGS . . . 290
13. DESIGN OF MEMBERS IN TENSION, BENDING, OR TORSION . . . 316
14. DESIGN OF COMPRESSION MEMBERS . . . 344
15. DESIGN OF WEBS IN SHEAR . . . 393
16. DEFLECTIONS OF STRUCTURES . . . 419
17. STATICALLY INDETERMINATE STRUCTURES . . . 454
18. SPECIAL METHODS OF ANALYSIS . . . 513
APPENDIX . . . 557
INDEX . . . 561

CHAPTER 1

EQUILIBRIUM OF FORCES

1.1. Equations of Equilibrium. One of the first steps in the design of a machine or structure is the determination of the loads acting on each member. The loads acting on an airplane may occur in various landing or flight conditions. The loads may be produced by ground reactions on the wheels, by aerodynamic forces on the wings and other surfaces, or by forces exerted on the propeller. The loads are resisted by the weight or inertia of the various parts of the airplane. Several loading conditions must be considered, and each member must be designed for the combination of conditions which produces the highest stress in the member. For practically all members of the airplane structure the maximum loads occur when the airplane is in an accelerated flight or landing condition and the external loads are not in equilibrium. If, however, the inertia loads are also considered, they will form a system of forces which are in equilibrium with the external loads. In the design of any member it is necessary to find all the forces acting on the member, including inertia forces. Where these forces are in the same plane, as is often the case, the following equations of static equilibrium apply to any isolated portion of the structure:

$$\left.\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M &= 0 \end{aligned}\right\} \qquad (1.1)$$

The terms ΣF_x and ΣF_y represent the summations of the components of forces along x and y axes, which may be taken in any two arbitrary directions. The term ΣM represents the sum of the moments of all forces about any arbitrarily chosen point in the plane. Each of these equations may be set up in an infinite number of ways for any problem, since the directions of the axes and the center of moments may be chosen arbitrarily. Only three independent equations exist for any free body, however, and only three unknown forces may be found from the equations. If, for example, an attempt is made to find four unknown forces by using the two force equations and moment equations about two points, the four equations cannot be solved because they are not independent, *i.e.*, one of the equations can be derived from the other three.

The following equations cannot be solved for the numerical values of the three unknowns because they are not independent.

$$x + y + z = 3$$
$$x + y + 2z = 4$$
$$2x + 2y + 3z = 7$$

The third equation may be obtained by adding the first two equations, and consequently does not represent an independent condition.

In the analysis of a structure containing several members it is necessary to draw a free-body diagram for each member, showing all the forces acting on that member. It is not possible to show these forces on a composite sketch of the entire structure, since equal and opposite forces act at all joints and an attempt to designate the correct direction of the force on each member will be confusing. In applying the equations of statics it is desirable to choose the axes and centers of moments so that only one unknown appears in each equation.

Many structural joints are made with a single bolt or pin. Such joints are assumed to have no resistance to rotation. The force at such a joint must pass through the center of the pin, as shown in Fig. 1.1, since the moment about the center of the pin must be zero. The force at the pin joint has two unknown quantities, the magnitude F and the direction θ. It is usually more convenient to find the two unknown components, F_x and F_y, from which F and θ can be found by the equations:

$$F = \sqrt{F_x^2 + F_y^2} \tag{1.2}$$

$$\tan \theta = \frac{F_y}{F_x} \tag{1.3}$$

The statics problem is considered as solved when the components F_x and F_y at each joint are obtained.

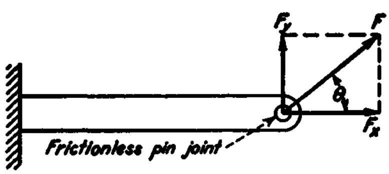


FIG. 1.1.

FIG. 1.2.

1.2. Two-force Members. When a structural member has forces acting at only two points, these forces must be equal and opposite, as shown in Fig. 1.2. Since moments about point A must be zero, the force F_B must pass through point A. Similarly the force F_A must pass through point B for moments about point B to be zero. From a summation of forces, the forces F_B and F_A must have equal magnitudes but opposite directions. Two-force members are frequently used in aircraft and other structures, since simple tension or compression members are usually the lightest members for transmitting forces. Where possible, two-force members are straight, rather than curved as shown in Fig. 1.2. Structures made up entirely of two-force members are called trusses and are frequently used in fuselages, engine mounts, and other aircraft structures, as well as in bridge and building structures. Trusses represent an important special type of structure and will be treated in detail in the following articles.

Many structures contain some two-force members, as well as some members which resist more than two forces. These structures must first be examined carefully in order to determine which members are two-force members. Students frequently make the serious mistake of assuming that forces act in the direction of a member, when the member resists forces at three or more points. In the case of curved two-force members such as shown in Fig. 1.2, it is important to assume that the forces act along the line between pins, rather than along the axis of the members.

While Eqs. 1.1 are simple and well known, it is very important for a student to acquire proficiency in the application of these equations to various types of structures. A typical structure will be analyzed as an example problem.

Example. Find the forces acting at all joints of the structure shown in Fig. 1.3.

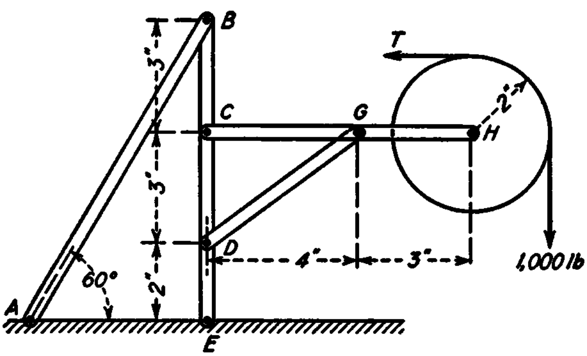


FIG. 1.3.

Solution. First draw free-body diagrams of all members, as shown in Fig. 1.4. Since AB and GD are two-force members, the forces in these members are along the line joining the pin joints of these members, and free-body diagrams of these members are not shown. The directions of forces are assumed, but care must be taken to show the forces at any joint in opposite directions on the two members, that is, C_x is assumed to act to the right on the horizontal member, and therefore must act to the left on the vertical member. If forces are assumed in the wrong direction, the calculated magnitudes will be negative.

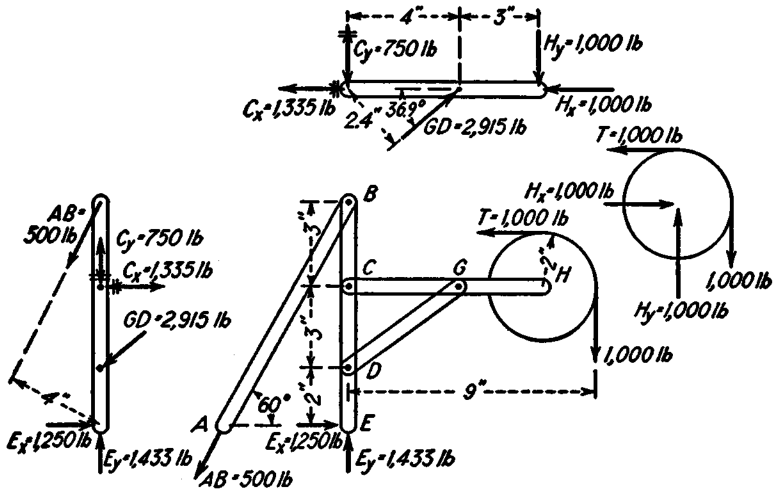


FIG. 1.4.

For the pulley,

$$\Sigma M_H = 2 \times 1{,}000 - 2T = 0$$
$$T = 1{,}000 \text{ lb}$$
$$\Sigma F_y = H_y - 1{,}000 = 0$$
$$H_y = 1{,}000 \text{ lb}$$
$$\Sigma F_x = H_x - 1{,}000 = 0$$
$$H_x = 1{,}000 \text{ lb}$$

Once the numerical values of these forces are obtained, they are shown on the free-body diagram. Subsequent equations contain the known numerical values rather than the algebraic symbols for the forces.

For member CGH,

$$\Sigma M_C = 1{,}000 \times 7 - 2.4GD = 0$$
$$GD = 2{,}915 \text{ lb}$$
$$\Sigma F_x = C_x + 2{,}915 \cos 36.9° - 1{,}000 = 0$$
$$C_x = -1{,}335 \text{ lb}$$
$$\Sigma F_y = C_y + 2{,}915 \sin 36.9° - 1{,}000 = 0$$
$$C_y = -\ 750 \text{ lb}$$

Since C_x and C_y are negative, the directions of the vectors on the free-body diagrams are changed. Such changes are made by crossing out the original arrows rather than by erasing, in order that the analysis may be checked conveniently by the original designer or by others. Extreme care must be observed in using the proper direction of known forces.

For member *BCDE*,

$$\Sigma M_E = 1{,}335 \times 5 - 2{,}915 \times 2 \cos 36.9° - 4.0AB = 0$$
$$AB = 500 \text{ lb}$$
$$\Sigma F_x = E_x - 2{,}915 \cos 36.9° + 1{,}335 - 500 \cos 60° = 0$$
$$E_x = 1{,}250 \text{ lb}$$
$$\Sigma F_y = E_y - 2{,}915 \sin 36.9° + 750 - 500 \sin 60° = 0$$
$$E_y = 1{,}433 \text{ lb}$$

All forces have now been obtained without the use of the entire structure as a free body. The solution will be checked by using all three equations of equilibrium for the entire structure.

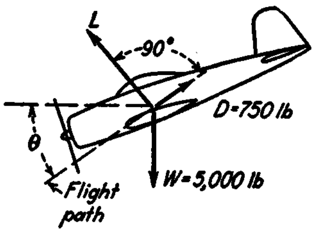
Check using entire structure as free body,

$$\Sigma F_x = 1{,}250 - 1{,}000 - 500 \cos 60° = 0$$
$$\Sigma F_y = 1{,}433 - 1{,}000 - 500 \sin 60° = 0$$
$$\Sigma M_E = 1{,}000 \times 9 - 1{,}000 \times 7 - 500 \times 4 = 0$$

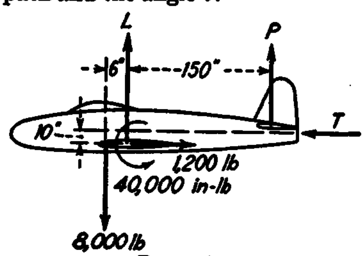
This check should be made wherever possible in order to detect errors in computing moment arms or forces.

PROBLEMS

1.1. A 5,000-lb airplane is in a steady glide with the flight path at an angle θ below the horizontal. The drag force in the direction of the flight path is 750 lb. Find the lift force L normal to the flight path and the angle θ.



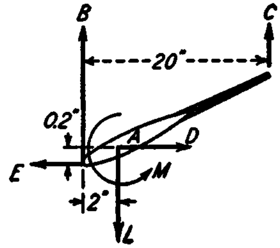
PROB. 1.1.



PROB. 1.2.

1.2. A jet-propelled airplane in steady flight has forces acting as shown. Find the jet thrust T, lift L, and tail load P.

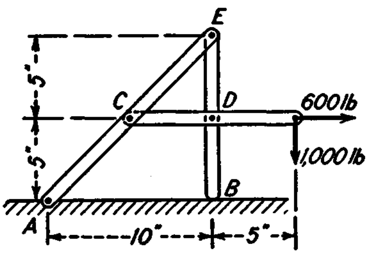
1.3. A wind-tunnel model of an airplane wing is suspended as shown. Find the loads in members B, C, and E if the forces at A are $L = 43.8$ lb, $D = 3.42$ lb, and $M = -20.6$ in-lb.



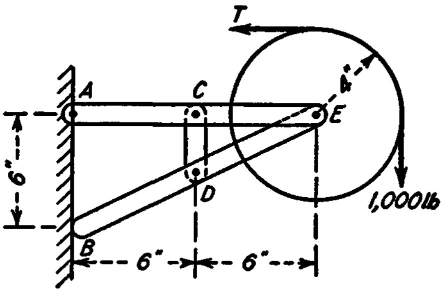
PROB. 1.3 and 1.4.

1.4. For the model of Prob. 1.3 find the forces L, D, and M at a point A, if the measured forces are $B = 40.2$ lb, $C = 4.16$ lb, and $E = 3.74$ lb.

1.5. Find the horizontal and vertical components of the forces at all joints. The reaction at point B is vertical. Check results by using the three remaining equations of statics.



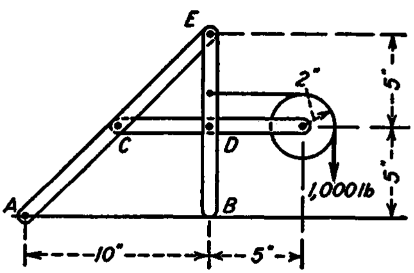
PROB. 1.5.



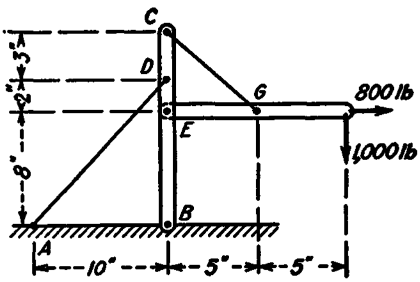
PROB. 1.6.

1.6. Find the horizontal and vertical components of the forces at all joints. The reaction at point B is horizontal. Check results by three equations.

1.7. Find the horizontal and vertical components of the forces at all joints. The reaction at point B is vertical. Check results by three equations.

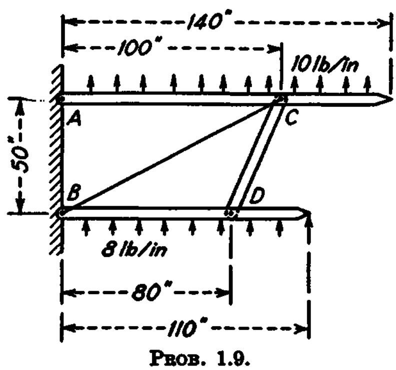


PROB. 1.7.



PROB. 1.8.

1.8. Find the forces at all joints of the structure. Check results by three equations.



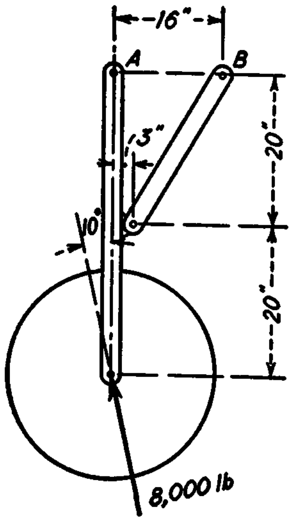
PROB. 1.9.

1.9. Find the forces on all members of the biplane structure shown. Check results by considering equilibrium of entire structure as a free body.

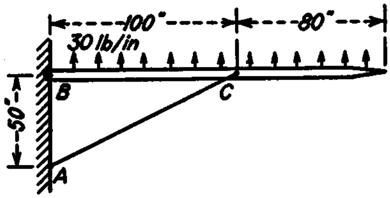
1.10. Find the forces at points A and B of the landing gear shown.

1.11. Find the forces at points A, B, and C of the structure of the braced-wing monoplane shown.

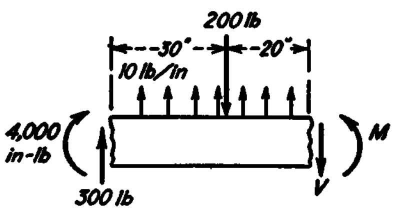
1.12. Find the forces V and M at the cut cross section of the beam.



PROB. 1.10.



PROB. 1.11.



PROB. 1.12.

1.3. Truss Structures. A truss has been defined as a structure which is composed entirely of two-force members. In some cases the members have a single bolt or pin connection at each end, and the external loads are applied only at the pin joints. In other cases the members are welded or riveted at the ends, but are assumed to be pin-connected in the analysis because it has been found that such an analysis yields approximately the correct values for the forces in the members. The trusses considered in this chapter are assumed to be coplanar; the loads resisted by the truss and the axes of all of the truss members lie in the same plane.

Trusses may be classified as statically determinate and statically indeterminate. The forces in all of the members of a statically determinate truss may be obtained from the equations of statics. In a statically indeterminate truss, there are more unknown forces than the number of independent equations of statics, and the forces cannot be determined from the equations of statics. If a rigid structure is supported in such a manner that three nonparallel, nonconcurrent reaction components are developed, the three reaction forces may be obtained from the three equations of statics for the entire structure as a free body. If more than three support reactions are developed, the structure is statically indeterminate externally.

Trusses which have only three reaction force components, but which contain more members than required, are statically indeterminate internally. Trusses are normally formed of a series of triangular frames. The first triangle contains three members and three joints. Additional triangles are each formed by adding two members and one joint. The number of members m has the following relationship to the number of joints j.

$$m - 3 = 2(j - 3)$$

or

$$m = 2j - 3 \tag{1.4}$$

If a truss has one less member than the number specified by Eq. 1.4, it becomes a linkage or mechanism, with one degree of freedom. A linkage is not capable of resisting loads, and is classified as an unstable structure. If a truss has one more member than the number specified by Eq. 1.4, it is statically indeterminate internally.

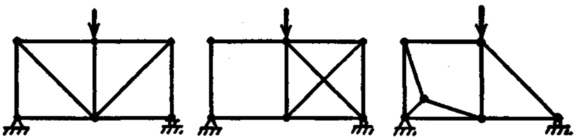
If each pin joint of a truss is considered as a free body, the two statics equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$ may be applied. The equation $\Sigma M = 0$ does not apply, since all forces act through the pin, and the moments about the pin will be zero regardless of the magnitudes of the forces. Thus, for a truss with j joints, there are $2j$ independent equations of statics. The equations for the equilibrium of the entire structure are not independent of the equations for the joints, since they can be derived from the equations of equilibrium for the joints. For example, the equation $\Sigma F_x = 0$ for the entire structure may be obtained by adding all the equations $\Sigma F_x = 0$ for the individual joints. The equations $\Sigma F_y = 0$ and $\Sigma M = 0$ for the entire truss may similarly be obtained from the equations for the joints. Equation 1.4 may therefore be derived in another manner by equating the number of unknown forces for m members and three reactions to the number of independent equations $2j$, or $m + 3 = 2j$.

It is necessary to apply Eq. 1.4 with care. The equation is applicable for the normal truss which contains a series of triangular frames and has three external reactions, such as the truss shown in Fig. 1.5(*a*). For other trusses it is necessary to determine by inspection that all parts of the structure are stable. The truss shown in Fig. 1.5(*b*) satisfies Eq. 1.4; yet the left panel is unstable, while the right panel has one more diagonal than is necessary. The truss shown in Fig. 1.5(*c*) is stable and statically determinate, even though it is not constructed entirely of triangular frames.

Some trusses may have more than three external reactions, and fewer members than are specified by Eq. 1.4, and be stable and statically determinate. The number of reactions r may be substituted for the three in Eq. 1.4.

$$m = 2j - r \tag{1.4a}$$

The number of independent equations, $2j$, is therefore sufficient to obtain the $m + r$ unknown forces for the members and the reactions. An example of a stable and statically determinate truss which has four reactions may be obtained from the truss of Fig. 1.5(*a*) by adding a horizontal reaction at the upper left-hand corner and removing the right-hand diagonal member.



(a) Stable and statically determinate. *(b) Left panel unstable. Right panel statically indeterminate.* *(c) Stable and statically determinate.*

All trusses have 9 members and 6 joints, or $m = 2j - 3$.

FIG. 1.5.

1.4. Truss Analysis by Method of Joints. In the analysis of a truss by the method of joints, the two equations of static equilibrium, $\Sigma F_x = 0$ and $\Sigma F_y = 0$, are applied for each joint as a free body. Two unknown forces may be obtained for each joint. Since each member is a two-force member, it exerts equal and opposite forces on the joints at its ends. The joints of a truss must be analyzed in sequence by starting at a joint which has only two members with unknown forces. After finding the forces in these two members, an adjacent joint at the end of one of these members will have only two unknown forces. The joints are then analyzed in the proper sequence until all joints have been considered.

In most structures it is necessary to determine the three external reactions from the equations of equilibrium for the entire structure, in order to have only two unknown forces at each joint. These three equations are used in addition to the $2j$ equations at the joints. Since there are only $2j$ unknown forces, three of the equations are not necessary for finding the unknowns, but should always be used for checking the numerical work. The analysis of a truss by the method of joints will be illustrated by a numerical example.

Example. Find the loads in all the members of the truss shown in Fig. 1.6.

Solution. Draw a free-body diagram for the entire structure and for each joint, as shown in Fig. 1.7. Since all loads in the two-force members act along the members, it is possible to show all forces on a sketch of the truss, as shown in Fig. 1.7, if the forces are specified as acting on the joints. Care must be used in the directions of the vectors, since at every point there is always a force acting on the member which is equal and opposite to the force acting on the joint. If a structure contains any members which are not two-force members, it is always necessary to make separate free-body sketches of these members, as shown in Fig. 1.4.

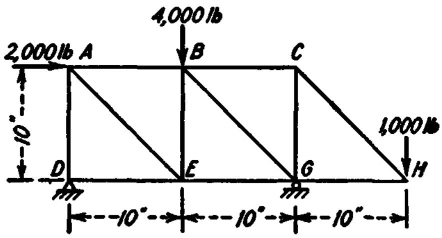


Fig. 1.6.

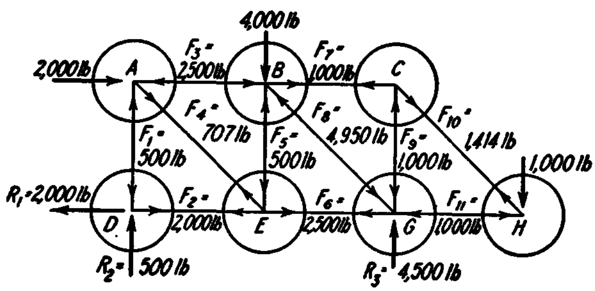


Fig. 1.7.

Considering the entire structure as a free body,

$$\Sigma M_D = 2{,}000 \times 10 + 4{,}000 \times 10 + 1{,}000 \times 30 - 20R_3 = 0$$
$$R_3 = 4{,}500 \text{ lb}$$
$$\Sigma F_y = R_2 - 4{,}000 - 1{,}000 + 4{,}500 = 0$$
$$R_2 = 500 \text{ lb}$$
$$\Sigma F_x = 2{,}000 - R_1 = 0$$
$$R_1 = 2{,}000 \text{ lb}$$

The directions of unknown forces are assumed, as in the previous example, and vectors changed on the sketch when they are found to be negative. Some engineers prefer to assume all members in tension, in which case negative signs designate compression. Joints must be selected in the proper order, so that there are only two unknowns at each joint.

Joint D:

$$\Sigma F_x = F_2 - 2{,}000 = 0$$
$$F_2 = 2{,}000 \text{ lb}$$
$$\Sigma F_y = 500 - F_1 = 0$$
$$F_1 = 500 \text{ lb}$$

Joint A:

$$\Sigma F_y = 500 - F_4 \sin 45° = 0$$
$$F_4 = 707 \text{ lb}$$
$$\Sigma F_x = 2{,}000 + 707 \cos 45° - F_3 = 0$$
$$F_3 = 2{,}500 \text{ lb}$$

Joint E:

$$\Sigma F_x = F_6 - 2{,}000 - 707 \cos 45° = 0$$
$$F_6 = 2{,}500 \text{ lb}$$
$$\Sigma F_y = 707 \sin 45° - F_5 = 0$$
$$F_5 = 500 \text{ lb}$$

Joint B:

$$\Sigma F_y = 500 - 4{,}000 + F_8 \sin 45° = 0$$
$$F_8 = 4{,}950 \text{ lb}$$
$$\Sigma F_x = 2{,}500 - 4{,}950 \cos 45° + F_7 = 0$$
$$F_7 = 1{,}000 \text{ lb}$$

Joint C:

$$\Sigma F_x = F_{10} \cos 45° - 1{,}000 = 0$$
$$F_{10} = 1{,}414 \text{ lb}$$
$$\Sigma F_y = F_9 - 1{,}414 \sin 45° = 0$$
$$F_9 = 1{,}000 \text{ lb}$$

Joint G:

$$\Sigma F_x = 4{,}950 \cos 45° - 2{,}500 - F_{11} = 0$$
$$F_{11} = 1{,}000 \text{ lb}$$

Check: $\Sigma F_y = 4{,}500 - 4{,}950 \cos 45° - 1{,}000 = 0$

Joint H:

Check: $\Sigma F_y = 1{,}414 \sin 45° - 1{,}000 = 0$

Check: $\Sigma F_x = 1{,}000 - 1{,}414 \cos 45° = 0$

Arrows acting toward a joint show that a member is in compression, and arrows acting away from a joint indicate tension.

1.5. Truss Analysis by Method of Sections. It is often desirable to find the forces in some of the members of a truss without analyzing the entire truss. The method of joints is usually cumbersome in this case, since the forces in all members to the left of any member must be obtained before finding the force in that particular member. An analysis by the method of sections will yield the force in any member by a single operation, without the necessity of finding the forces in the other members. Instead of considering the joints as free bodies, a cross section is taken through the truss, and the part of the truss on one side of the cross section is considered as a free body. The cross section is chosen so that it cuts the members for which the forces are desired and so that it preferably cuts only three members.

If the forces in members *BC*, *BG*, and *EG* of the truss of Fig. 1.7 are desired, the free body will be as shown in Fig. 1.8. The three unknowns may be found from the three equations for static equilibrium.

$$\Sigma M_G = 10F_7 - 10 \times 4{,}000 + 2{,}000 \times 10 + 500 \times 20 = 0$$
$$F_7 = 1{,}000 \text{ lb}$$
$$\Sigma F_y = F_8 \sin 45° - 4{,}000 + 500 = 0$$
$$F_8 = 4{,}950 \text{ lb}$$
$$\Sigma F_x = F_6 + 1{,}000 - 4{,}950 \cos 45° + 2{,}000 - 2{,}000 = 0$$
$$F_6 = 2{,}500 \text{ lb}$$

These values check those obtained in the analysis by the method of joints.

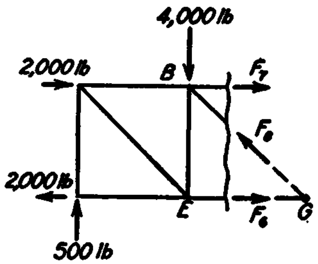


FIG. 1.8.

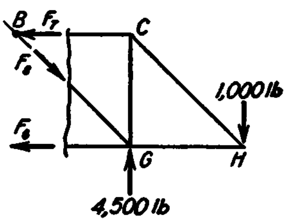


FIG. 1.9.

The portion of the truss to the right of the section through the members might have been taken as the free body, as shown in Fig. 1.9. The equations of equilibrium would be as follows:

$$\Sigma M_G = 1{,}000 \times 10 - 10F_7 = 0$$
$$F_7 = 1{,}000 \text{ lb}$$
$$\Sigma F_y = 4{,}500 - 1{,}000 - F_8 \sin 45° = 0$$
$$F_8 = 4{,}950 \text{ lb}$$
$$\Sigma F_x = 4{,}950 \cos 45° - 1{,}000 - F_6 = 0$$
$$F_6 = 2{,}500 \text{ lb}$$

It would also be possible to find the force F_6 by taking moments about point *B*, thus eliminating the necessity of first finding the forces F_7 and F_8.

1.6. Truss Analysis—Graphic Method. In the analysis of trusses by the method of joints, two unknown forces were obtained from the equations of equilibrium. It is also possible to find two unknown forces at each joint graphically by the use of the force polygon for the joint. The joints must be analyzed in the same sequence as used in the method of joints. In the graphic truss analysis, it is convenient to use Bow's notation, in which each space is designated by a letter and forces are designated by the two letters corresponding to the spaces on each side of the force. The truss of Fig. 1.6 will be analyzed graphically, and the notation used will be as shown in Fig. 1.10. The capital letters designating the joints are usually omitted but are included here only for reference during the discussion.

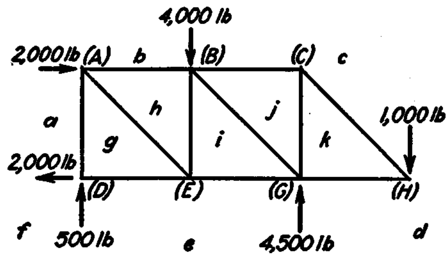


FIG. 1.10.

The external reactions can be determined graphically, but since it is usually more convenient to use algebraic methods the graphic solution will not be considered here. Using the external reactions found in Art. 1.4, a force polygon for the entire structure as a free body is shown in Fig. 1.11(*a*). While *ab* and *fa* are on the same horizontal line, and *bc* and *de* are on the same vertical line and will be shown that way in future work, they are shown displaced slightly for purposes of explanation. The notation shown is such that when the letters are read clockwise around the structure of Fig. 1.10, *a*, *b*, *c*, *d*, *e*, and *f*, the directions of the forces in the force polygon will be *a* to *b*, *b* to *c*, *c* to *d*, *d* to *e*, and *e* to *f*, with the polygon closing by the force *fa*.

Joint *D* is first considered as a free body and the known forces *ef* and *fa* drawn to scale. The unknown forces *ag* and *ge* are then drawn in the proper directions from *a* and *e*, and the magnitudes are determined from the intersection *g*, as shown in Fig. 1.11(*b*). Joint *A* is next analyzed by drawing known forces *ga* and *ab* and finding *bh* and *hg* by the intersection at *h* in Fig. 1.11(*c*). Similarly, joints *E*, *B*, *C*, and *G* are analyzed as shown in Figs. 1.11(*d*) to (*g*). All forces have now been determined without considering joint *H*. The force polygon for joint *H*, shown in Fig. 1.11(*h*), is used for checking results, as in the algebraic solution.

A study of the force polygons shows that each force appears in two polygons. Force *ge* is a force to the right on joint *D*, and *eg* is a force to the left on joint *E*, but points *e* and *g* have the same relative position in both diagrams. All the polygons can be combined into one stress diagram as shown in Fig. 1.11(*i*). Arrows are omitted from the stress diagram, since there would always be forces in both directions and the arrows would have no significance. To determine the direction of the forces at any joint, the letters are read clockwise around the joint. Thus, at joint D in Fig. 1.10, the letters ge are read, which are seen in Fig. 1.11(i) to represent a force to the right on joint D. Proceeding

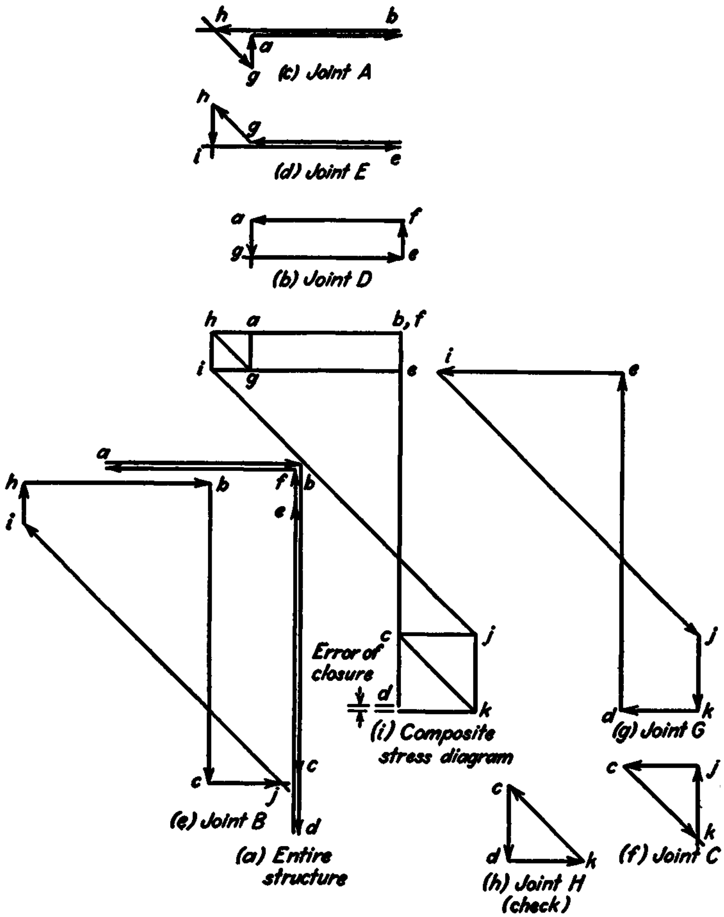


FIG. 1.11.

clockwise around joint E, the letters eg are read, which represent a force to the left in the stress diagram. The member is therefore a tension member exerting a force to the left on joint E.

Example. Construct a stress diagram for the steel-tube fuselage truss shown in Fig. 1.12. The structure is stable and statically determinate although space c is not triangular. A lighter structure would be obtained if a single diagonal were used in place of members ce, cd, and de, but this would not permit enough space for a side door in the fuselage. The type of framing shown is often used to permit openings in the truss for access purposes.

Solution. The reactions R_1 and R_2 are first obtained algebraically. The stress diagram is then constructed as shown in Fig. 1.12(*b*). The forces in all members are then obtained by scaling the lengths from the stress diagram. The

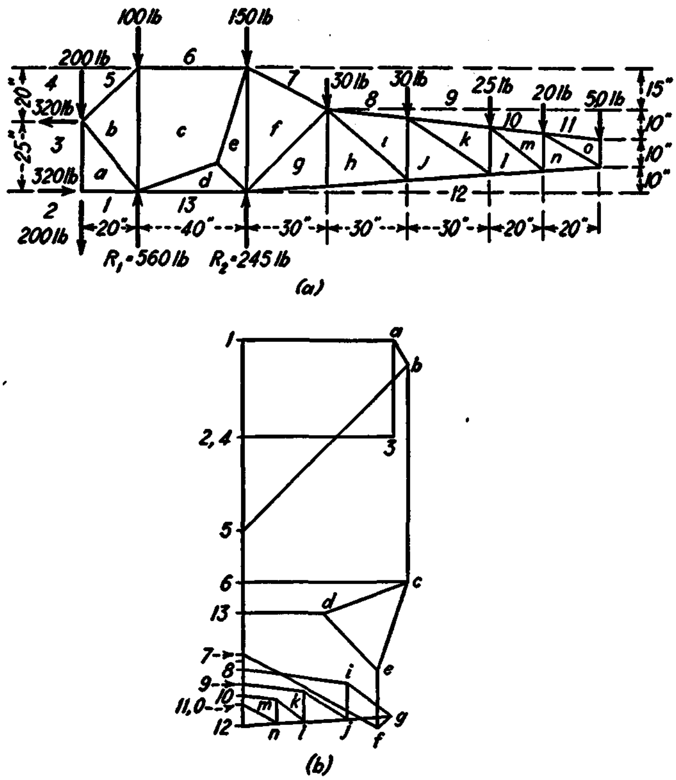


FIG. 1.12.

directions of the forces are obtained from the stress diagram in the same manner as in the previous example. For member 8–*i*, the line in the stress diagram is from left to right, indicating a tension force acting to the right on the joint to the left of the member. Reading clockwise around the joint to the right of the member, the force is *i*–8, which in the stress diagram is a force to the left. This checks the conclusion that the member is in tension.

An algebraic solution of problems such as the fuselage truss may become rather tedious, since each of the inclined members is at a different angle. The graphic solution has many advantages for a problem of this type, because it is much easier to draw the truss to scale and to project lines parallel to the members than to calculate the angles and forces for the members.

1.7. Trusses Containing Members in Bending. Many structures are made up largely of two-force members but contain some members which are loaded laterally, as shown in Fig. 1.13. These structures are usually classed as trusses, since the analysis is similar to that used for trusses. The horizontal members of the truss shown in Fig. 1.13 are

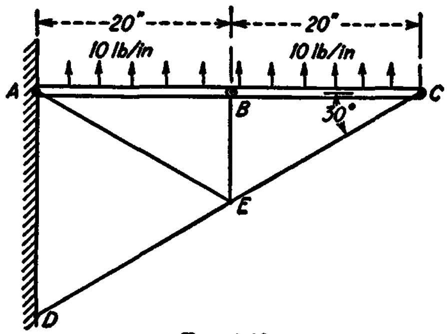


FIG. 1.13.

not two-force members, and separate free-body diagrams for these members, as shown in Figs. 1.14(*a*) and (*b*), are required. Since each of these members has four unknown reactions, the equations of statics are not sufficient for finding all four forces. It is possible to find the vertical forces $A_y = B_{y1} = B_{y2} = C_y = 100$ lb and to obtain the relations $A_x = B_{x1}$ and $B_{x2} = C_x$ from the equilibrium equations for the horizontal members.

When the forces obtained from the horizontal members are applied to the remaining structure as a free body, as shown in Fig. 1.14(*c*), it is apparent that the remaining structure may be analyzed by the same methods that were used in the previous truss problems. The loads obtained by such an analysis are shown in Fig. 1.14(*d*). All members except the horizontal members may now be designed as simple tension or compression members. The horizontal members must be designed for bending moments combined with the compression load of 173.2 lb.

In the trusses previously analyzed, the members themselves have been assumed to be weightless. The effects of the weight of the members may be considered by the method used in the preceding example. It will be noticed that the correct axial loads in the truss members may be obtained if half the weight of the member is applied at each of the panel points at the ends of the member. The bending stresses in the member resulting from the weight of the member must be computed separately and combined with the axial stresses in the member.