AMSCO'S

ALGEBRA 2 and TRIGONOMETRY

Ann Xavier Gantert

AMSCO

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Dedication

To Jessica Alexander and Uriel Avalos in gratitude for their invaluable work in preparing this text for publication.

Ann Xavier Gantert

The author has been associated with mathematics education in New York State as a teacher and an author throughout the many changes of the past fifty years. She has worked as a consultant to the Mathematics Bureau of the Department of Education in the development and writing of Sequential Mathematics and has been a coauthor of Amsco's *Integrated Mathematics* series, which accompanied that course of study.

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PREFACE

Algebra 2 and Trigonometry is a new text for a course in intermediate algebra and trigonometry that continues the approach that has made Amsco a leader in presenting mathematics in a modern, integrated manner. Over the last decade, this approach has undergone numerous changes and refinements to keep pace with ever-changing technology.

This textbook is the final book in the three-part series in which Amsco parallels the integrated approach to the teaching of high school mathematics promoted by the National Council of Teachers of Mathematics in its *Principles and Standards for School Mathematics* and mandated by the New York State Board of Regents in the *Mathematics Core Curriculum*. The text presents a range of materials and explanations that are guidelines for achieving a high level of excellence in their understanding of mathematics.

In this book:

✔ **The real numbers** are reviewed and the understanding of operations with irrational numbers, particularly radicals, is expanded.

✔ **The graphing calculator** continues to be used as a routine tool in the study of mathematics. Its use enables the student to solve problems that require computation that more realistically reflects the real world. The use of the calculator replaces the need for tables in the study of trigonometry and logarithms.

✔ **Coordinate geometry** continues to be an integral part of the visualization of algebraic and trigonometric relationships.

✔ **Functions** represent a unifying concept throughout. The algebraic functions introduced in *Integrated Algebra 1* are reviewed, and exponential, logarithmic, and trigonometric functions are presented.

✔ **Algebraic skills** from *Integrated Algebra 1* are maintained, strengthened, and expanded as both a holistic approach to mathematics and as a bridge to advanced studies.

✔ **Statistics** includes the use of the graphing calculator to reexamine range, quartiles, and interquartile range, to introduce measures of dispersion such as variance and standard deviation, and to determine the curve that best represents a set of bivariate data.

✔ **Integration** of geometry, algebra, trigonometry, statistics, and other branches of mathematics begun in *Integrated Algebra 1* and *Geometry* is continued and further expanded.

✔ **Exercises** are divided into three categories. *Writing About Mathematics* encourages the student to reflect on and justify mathematical conjectures, to discover counterexamples, and to express mathematical ideas in his or her own words. *Developing Skills* provides routine practice exercises that enable the student and teacher to evaluate the student's ability to both manipulate mathematical symbols and understand mathematical relationships. *Applying Skills* provides exercises in which the new ideas of each section, together with previously learned skills, are used to solve problems that reflect real-life situations.

✔ **Problem solving**, a primary goal of all learning standards, is emphasized throughout the text. Students are challenged to apply what has been learned to the solution of both routine and non-routine problems.

✔ **Enrichment** is stressed both in the text and in the Teacher's Manual where many suggestion are given for teaching strategies and alternative assessment. The Manual provides opportunities for extended tasks and hands-on activities. Reproducible *Enrichment Activities* that challenge students to explore topics in greater depth are provided in each chapter of the Manual.

In this text, the real number system is expanded to include the complex numbers, and algebraic, exponential, logarithmic, and trigonometric functions are investigated. The student is helped to understand the many branches of mathematics, to appreciate the common threads that link these branches, and to recognize their interdependence.

The intent of the author is to make this book of greatest service to the average student through detailed explanations and multiple examples. Each section provides careful step-by-step procedures for solving routine exercises as well as the non-routine applications of the material. Sufficient enrichment material is included to challenge students of all abilities.

Specifically:

✔ Concepts are carefully developed using appropriate language and mathematical symbolism. General principles are stated clearly and concisely.

✔ Numerous examples serve as models for students with detailed explanations of the mathematical concepts that underlie the solution. Alternative approaches are suggested where appropriate.

✔ Varied and carefully graded exercises are given in abundance to develop skills and to encourage the application of those skills. Additional enrichment materials challenge the most capable students.

This text is offered so that teachers may effectively continue to help students to comprehend, master, and enjoy mathematics as they progress in their education.

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CHAPTER

1

THE INTEGERS

In golf tournaments, a player's standing after each hole is often recorded on the leaderboard as the number of strokes above or below a standard for that hole called a *par*. A player's standing is a positive number if the number of strokes used was greater than par and a negative number if the number of strokes used was less than par. For example, if par for the first hole is 4 strokes and a player uses only 3, the player's standing after playing the first hole is -1.

Rosie Barbi is playing in an amateur tournament. Her standing is recorded as 2 below par (-2) after sixteen holes. She shoots 2 below par on the seventeenth hole and 1 above par on the eighteenth. What is Rosie's standing after eighteen holes? Nancy Taylor, who is her closest opponent, has a standing of 1 below par (-1) after sixteen holes, shoots 1 below par on the seventeenth hole and 1 below par on the eighteenth. What is Nancy's standing after eighteen holes?

In this chapter, we will review the set of integers and the way in which the integers are used in algebraic expressions, equations, and inequalities.

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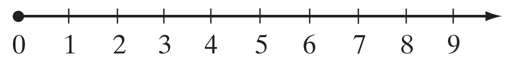
1-1 WHOLE NUMBERS, INTEGERS, AND THE NUMBER LINE

The first numbers that we learned as children and probably the first numbers used by humankind are the natural numbers. Most of us began our journey of discovery of the mathematical world by *counting*, the process that lists, in order, the names of the **natural numbers** or the **counting numbers**. When we combine the natural numbers with the number 0, we form the set of **whole numbers**:

$$\{0, 1, 2, 3, 4, 5, 6, \ldots\}$$

These numbers can be displayed as points on the number line:



The number line shows us the order of the whole numbers; 5 is to the right of 2 on the number line because $5 > 2$, and 3 is to the left of 8 on the number line because $3 < 8$. The number 0 is the smallest whole number. There is no largest whole number.

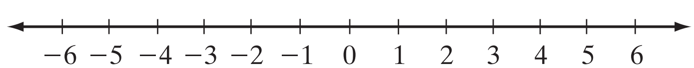
The temperature on a winter day may be two degrees above zero or two degrees below zero. The altitude of the highest point in North America is 20,320 feet above sea level and of the lowest point is 282 feet below sea level. We represent numbers less than zero by extending the number line to the left of zero, that is, to numbers that are less than zero, and by assigning to every whole number a an opposite, $-a$, such that $a + (-a) = 0$.

DEFINITION

The **opposite** or **additive inverse** of a is $-a$, the number such that

$$a + (-a) = 0.$$

The set of **integers** is the union of the set of whole numbers and their opposites. The set of non-zero whole numbers is the positive integers and the opposites of the positive integers are the negative integers.



Let a, b, and c represent elements of the set of integers. Under the operation of addition, the following properties are true:

1. Addition is closed: $a + b$ is an integer
2. Addition is commutative: $a + b = b + a$
3. Addition is associative: $(a + b) + c = a + (b + c)$
4. Addition has an identity element, 0: $a + 0 = a$
5. Every integer has an inverse: $a + (-a) = 0$

We say that the integers form a **commutative group** under addition because the five properties listed above are true for the set of integers.

Subtraction

DEFINITION

$a - b = c$ if and only if $b + c = a$.

Solve the equation $b + c = a$ for c:

$$b + c = a$$
$$-b + b + c = a + (-b)$$
$$c = a + (-b)$$

Therefore, $a - b = a + (-b)$.

Absolute Value

A number, a, and its opposite, $-a$, are the same distance from zero on the number line. When that distance is written as a positive number, it is called the **absolute value** of a.

- If $a > 0$, then $|a| = a - 0 = a$
- If $a < 0$, then $|a| = 0 - a = -a$

Note: When $a < 0$, a is a negative number and its opposite, $-a$, is a positive number.

For instance, $5 > 0$. Therefore, $|5| = 5 - 0 = 5$.

$-5 < 0$. Therefore, $|-5| = 0 - (-5) = 5$.

We can also say that $|a| = |-a| = a$ or $-a$, whichever is positive.

EXAMPLE 1

Show that the opposite of $-b$ is b.

Solution The opposite of b, $-b$, is the number such that $b + (-b) = 0$.

Since addition is commutative, $b + (-b) = (-b) + b = 0$.

The opposite of $-b$ is the number such that $(-b) + b = 0$. Therefore, the opposite of $-b$ is b.

Exercises

Writing About Mathematics

1. Tina is three years old and knows how to count. Explain how you would show Tina that $3 + 2 = 5$.

2. Greg said that $|a - b| = |b - a|$. Do you agree with Greg? Explain why or why not.

Developing Skills

In 3–14, find the value of each given expression.

3. $|6|$

4. $|-12|$

5. $|8 - 3|$

6. $|3 - 8|$

7. $|5 + (-12)|$

8. $|-12 + (-(-5))|$

9. $|4 - 6 + (-2)|$

10. $|8 + (10 - 18)|$

11. $|3| - |3|$

12. $|8| - |-2| - |2|$

13. $-(|-2| + |3|)$

14. $|4 - 3| + |-1|$

In 15–18, use the definition of subtraction to write each subtraction as a sum.

15. $8 - 5 = 3$

16. $7 - (-2) = 9$

17. $-2 - 5 = -7$

18. $-8 - (-5) = -3$

19. Two distinct points on the number line represent the numbers a and b. If $|5 - a| = |5 - b| = 6$, what are the values of a and b?

Applying Skills

In 20–22, Mrs. Menendez uses computer software to record her checking account balance. Each time that she makes an entry, the amount that she enters is added to her balance.

20. If she writes a check for \$20, how should she enter this amount?

21. Mrs. Menendez had a balance of \$52 in her checking account and wrote a check for \$75.

a. How should she enter the \$75?

b. How should her new balance be recorded?

22. After writing the \$75 check, Mrs. Menendez realized that she would be overdrawn when the check was paid by the bank so she transferred \$100 from her savings account to her checking account. How should the \$100 be entered in her computer program?

1-2 WRITING AND SOLVING NUMBER SENTENCES

Equations

A sentence that involves numerical quantities can often be written in the symbols of algebra as an equation. For example, let x represent any number. Then the sentence "Three less than twice a number is 15" can be written as:

$$2x - 3 = 15$$

When we translate from one language to another, word order often must be changed in accordance with the rules of the language into which we are translating. Here we must change the word order for "three less than twice a number" to match the correct order of operations.

The **domain** is the set of numbers that can replace the variable in an algebraic expression. A number from the domain that makes an equation true is a **solution** or **root** of the equation. We can find the solution of an equation by writing a sequence of **equivalent equations**, or equations that have the same solution set, until we arrive at an equation whose solution set is evident. We find equivalent equations by changing both sides of the given equation in the same way. To do this, we use the following properties of equality:

Properties of Equality

- **Addition Property of Equality:** If equals are added to equals, the sums are equal.
- **Subtraction Property of Equality:** If equals are subtracted from equals, the differences are equal.
- **Multiplication Property of Equality:** If equals are multiplied by equals, the products are equal.
- **Division Property of Equality:** If equals are divided by non-zero equals, the quotients are equal.

On the left side of the equation $2x - 3 = 15$, the variable is multiplied by 2 and then 3 is subtracted from the product. We will simplify the left side of the equation by "undoing" these operations in reverse order, that is, we will first add 3 and then divide by 2. We can check that the number we found is a root of the given equation by showing that when it replaces x, it gives us a correct statement of equality.

$$\begin{aligned} 2x - 3 &= 15 \\ 2x - 3 + 3 &= 15 + 3 \\ 2x &= 18 \\ x &= 9 \end{aligned}$$

Check

$$\begin{aligned} 2x - 3 &= 15 \\ 2(9) - 3 &\stackrel{?}{=} 15 \\ 15 &= 15 \checkmark \end{aligned}$$

Often the definition of a mathematical term or a formula is needed to write an equation as the following example demonstrates:

EXAMPLE 1

Let $\angle A$ be an angle such that the complement of $\angle A$ is 6 more than twice the measure of $\angle A$. Find the measure of $\angle A$ and its complement.

Solution To write an equation to find $\angle A$, we must know that two angles are complements if the sum of their measures is 90°.

Let x = the measure of $\angle A$.

Then $2x + 6$ = the measure of the complement of $\angle A$.

The sum of the measures of an angle and of its complement is 90.

$$\begin{aligned} x + 2x + 6 &= 90 \\ 3x + 6 &= 90 \\ 3x &= 84 \\ x &= 28 \\ 2x + 6 &= 2(28) + 6 = 62 \end{aligned}$$

Therefore, the measure of $\angle A$ is 28 and the measure of its complement is 62.

Check The sum of the measures of $\angle A$ and its complement is 28 + 62 or 90. ✔

Answer $m\angle A = 28$; the measure of the complement of $\angle A$ is 62.

EXAMPLE 2

Find the solution of the following equation: $|6x - 3| = 15$.

Solution Since $|15| = |-15| = 15$, the algebraic expression $6x - 3$ can be equal to 15 or to −15.

$$\begin{aligned} 6x - 3 &= 15 \\ 6x - 3 + 3 &= 15 + 3 \\ 6x &= 18 \\ x &= 3 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 6x - 3 &= -15 \\ 6x - 3 + 3 &= -15 + 3 \\ 6x &= -12 \\ x &= -2 \end{aligned}$$

Check: $x = 3$

$$\begin{aligned} |6x - 3| &= 15 \\ |6(3) - 3| &\stackrel{?}{=} 15 \\ |15| &= 15 \checkmark \end{aligned}$$

Check: $x = -2$

$$\begin{aligned} |6x - 3| &= 15 \\ |6(-2) - 3| &\stackrel{?}{=} 15 \\ |-15| &= 15 \checkmark \end{aligned}$$

Answer The solution set is $\{3, -2\}$.

Inequalities

A number sentence can often be an inequality. To find the solution set of an inequality, we use methods similar to those that we use to solve equations. We need the following two properties of inequality:

Properties of Inequality

- **Addition and Subtraction Property of Inequality:** If equals are added to or subtracted from unequals, the sums or differences are unequal in the same order.
- **Multiplication and Division Property of Inequality:** If unequals are multiplied or divided by positive equals, the products or quotients are unequal in the same order. If unequals are multiplied or divided by negative equals, the products or quotients are unequal in the opposite order.

EXAMPLE 3

Find all positive integers that are solutions of the inequality $4n + 7 < 27$.

Solution We solve this inequality by using a procedure similar to that used for solving an equation.

$$4n + 7 < 27$$
$$4n + 7 + (-7) < 27 + (-7)$$
$$4n < 20$$
$$n < 5$$

Since n is a positive integer, the solution set is $\{1, 2, 3, 4\}$. *Answer*

EXAMPLE 4

Polly has \$210 in her checking account. After writing a check for tickets to a concert, she has less than \$140 in her account but she is not overdrawn. If each ticket cost \$35, how many tickets could she have bought?

Solution Let x = the number of tickets.

The cost of x tickets, $35x$, will be subtracted from \$210, the amount in her checking account. Since she is not overdrawn after writing the check, her balance is at least 0 and less than \$140.

$$0 \leq 210 - 35x < 140$$

$$\underline{-210} \quad \underline{-210 \qquad} \quad \underline{-210}$$ Add −210 to each member of the inequality.

$$-210 \leq -35x < -70$$

$$\frac{-210}{-35} \geq \frac{-35x}{-35} > \frac{-70}{-35}$$ Divide each member of the inequality by −35.

$$6 \geq x > 2$$ Note that dividing by a negative number reverses the order of the inequality.

Polly bought more than 2 tickets but at most 6.

Answer Polly bought 3, 4, 5, or 6 tickets.

Exercises

Writing About Mathematics

1. Explain why the solution set of the equation $12 - |x| = 15$ is the empty set.
2. Are $-4x > 12$ and $x > -3$ equivalent inequalities? Justify your answer.

Developing Skills

In 3–17, solve each equation or inequality. Each solution is an integer.

3. $5x + 4 = 39$
4. $7x + 18 = 39$
5. $3b + 18 = 12$
6. $12 - 3y = 18$
7. $9a - 7 = 29$
8. $13 - x = 15$
9. $|2x + 4| = 22$
10. $|3 - y| = 8$
11. $|4a - 12| = 16$
12. $|2x + 3| - 8 = 15$
13. $7a + 3 > 17$
14. $9 - 2b \leq 1$
15. $3 < 4x - 1 < 11$
16. $0 < x - 3 < 4$
17. $5 \geq 4b + 9 \geq 17$

Applying Skills

In 18–23, write and solve an equation or an inequality to solve the problem.

18. Peter had 156 cents in coins. After he bought 3 packs of gum he had no more than 9 cents left. What is the minimum cost of a pack of gum?

19. In an algebra class, 3 students are working on a special project and the remaining students are working in groups of five. If there are 18 students in class, how many groups of five are there?

20. Andy paid a reservation fee of \$8 plus \$12 a night to board her cat while she was on vacation. If Andy paid \$80 to board her cat, how many nights was Andy on vacation?

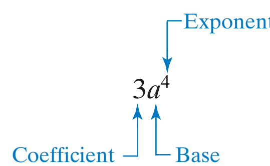
21. At a parking garage, parking costs \$5 for the first hour and \$3 for each additional hour or part of an hour. Mr. Kanesha paid \$44 for parking on Monday. For how many hours did Mr. Kanesha park his car?

22. Kim wants to buy an azalea plant for \$19 and some delphinium plants for \$5 each. She wants to spend less than \$49 for the plants. At most how many delphinium plants can she buy?

23. To prepare for a tennis match and have enough time for schoolwork, Priscilla can practice no more than 14 hours.. If she practices the same length of time on Monday through Friday, and then spends 4 hours on Saturday, what is the most time she can practice on Wednesday?

1-3 ADDING POLYNOMIALS

A **monomial** is a constant, a variable, or the product of constants and variables. Each algebraic expression, 3, a, ab, $-2a^2$, is a monomial.



A **polynomial** is the sum of monomials. Each monomial is a **term** of the polynomial. The expressions $3a^2 + 7a - 2$ is a polynomial over the set of integers since all of the numerical coefficients are integers. For any integral value of a, $3a^2 + 7a - 2$ has an integral value. For example, if $a = -2$, then:

$$\begin{aligned} 3a^2 + 7a - 2 &= 3(-2)^2 + 7(-2) - 2 \\ &= 3(4) + 7(-2) - 2 \\ &= 12 - 14 - 2 \\ &= -4 \end{aligned}$$

The same properties that are true for integers are true for polynomials: we can use the commutative, associative, and distributive properties when working with polynomials. For example:

$$\begin{aligned} (3a^2 + 5a) + (6 - 7a) &= (3a^2 + 5a) + (-7a + 6) && \text{Commutative Property} \\ &= 3a^2 + (5a - 7a) + 6 && \text{Associative Property} \\ &= 3a^2 + (5 - 7)a + 6 && \text{Distributive Property} \\ &= 3a^2 - 2a + 6 \end{aligned}$$

Note: When the two polynomials are added, the two terms that have the same power of the same variable factor are combined into a single term.

Two terms that have the same variable and exponent or are both numbers are called **similar terms** or **like terms**. The sum of similar terms is a monomial.

$3a^2 + 5a^2$	$-7ab + 3ab$	$x^3 + 4x^3$
$= (3 + 5)a^2$	$= (-7 + 3)ab$	$= (1 + 4)x^3$
$= 8a^2$	$= -4ab$	$= 5x^3$

Two monomials that are not similar terms cannot be combined. For example, $4x^3$ and $3x^2$ are not similar terms and the sum $4x^3 + 3x^2$ is not a monomial. A polynomial in simplest form that has two terms is a **binomial**. A polynomial in simplest form that has three terms is a **trinomial**.

Solving Equations and Inequalities

An equation or inequality often has a variable term on both sides. To solve such an equation or inequality, we must first write an equivalent equation or inequality with the variable on only one side.

For example, to solve the inequality $5x - 7 > 3x + 9$, we will first write an equivalent inequality that does not have a variable in the right side. Add the opposite of $3x$, $-3x$, to both sides. The terms $3x$ and $-3x$ are similar terms whose sum is 0.

$5x - 7 > 3x + 9$	
$-3x + 5x - 7 > -3x + 3x + 9$	Add $-3x$, the opposite of $3x$, to both sides.
$2x - 7 > 9$	$-3x + 3x = (-3 + 3)x = 0x = 0$
$2x - 7 + 7 > 9 + 7$	Add 7, the opposite of -7, to both sides.
$2x > 16$	Divide both sides by 2. Dividing by a positive does not reverse the inequality.
$x > 8$	

If x is an integer, then the solution set is $\{9, 10, 11, 12, 13, \ldots\}$.

EXAMPLE 1

a. Find the sum of $x^3 - 5x + 9$ and $x - 3x^3$.

b. Find the value of each of the given polynomials and the value of their sum when $x = -4$.

Solution **a.** The commutative and associative properties allow us to change the order and the grouping of the terms.