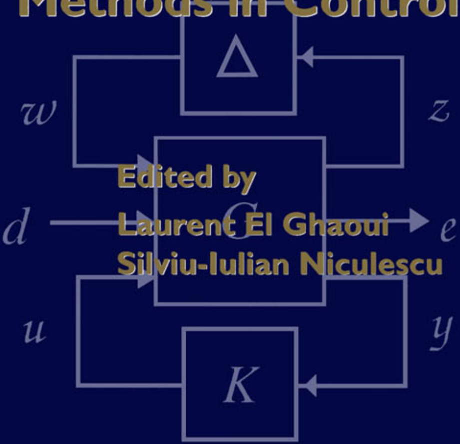
Advances in Linear Matrix Inequality Methods in Control



Edited by

Laurent El Ghaoui
Silviu-Iulian Niculescu

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Advances in Linear Matrix Inequality Methods in Control

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Advances in Linear Matrix Inequality Methods in Control

Edited by
Laurent El Ghaoui
University of California, Berkeley

Silviu-Iulian Niculescu
Université de Technologie de Compiègne
Compiègne, France

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Contributors

R. J. Adams	Electrical Engineering Department, University of Washington, Seattle, WA 98195 Internet: `rjadams@uwashington.edu`
C. Andriot	Service Téléopération et Robotique, CEA, route du Panorama, 92 265 Fontenay aux Roses, France Internet: `andriot@CYBORG.cea.fr`
P. Apkarian	CERT/DERA, 2 Av. Ed. Belin, 31055 Toulouse, France Internet: `apkarian@cert.fr`
V. Balakrishnan	School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47907-1285 Internet: `ragu@ecn.purdue.edu`
E. B. Beran	Axapta Financials Team, Damgaard International A/S Bregneroedvej 133, DK-3460 Birkeroed, Denmark Internet: `ebb@dk.damgaard.com`
S. Boyd	Information Systems Laboratory, Electrical Engineering Department, Stanford University, Stanford, CA 94305 Internet: `boyd@stanford.edu`
S. Dasgupta	Department of Electrical and Computer Engineering, University of Iowa, Iowa City, IA 52242 Internet: `soura-dasgupta@uiowa.edu`
C. E. de Souza	Laboratório Nacional de Computação Científica–LNCC, Department of Systems and Control, Av. Getulio Vargas, 333, 25651-070 Petrópolis, RJ, Brazil Internet: `csouza@lncc.br`
S. Dussy	Aérospatiale-Matra Lanceurs 66, route de Verneuil, 78133 Les Mureaux Cédex, France Internet: `stephane.dussy@espace.aerospatiale.fr`
L. El Ghaoui	Laboratoire de Mathématiques Appliquées, ENSTA, 32, Bvd. Victor, 75739 Paris Cédex 15, France Internet: `elghaoui@ensta.fr`
E. Feron	Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA 02139 Internet: `feron@mit.edu`

J. P. Folcher	Laboratoire de Mathématiques Appliquées, ENSTA, 32, Bvd. Victor, 75739 Paris Cédex 15, France Internet: `folcher@ensta.fr`
M. Fu	Department of Electrical and Computer Engineering, The University of Newcastle, Newcastle, NSW 2308, Australia Internet: `eemf@ee.newcastle.edu.au`
K. M. Grigoriadis	Department of Mechanical Engineering, University of Houston, Houston, TX 77204 Internet: `karolos@uh.edu`
J.-P. A. Haeberly	Mathematics Department, Fordham University, Bronx, NY 10458 Internet: `haeberly@murray.fordham.edu`
S. R. Hall	Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA 02139 Internet: `hall@mit.edu`
T. Iwasaki	Department of Control Systems Engineering, Tokyo Institute of Technology, 2-12-1 Oookayama, Meguro-ku, Tokyo 152, Japan Internet: `iwasaki@ctrl.titech.ac.jp`
U. Jönsson	Division of Optimization and Systems Theory, Department of Mathematics, Royal Institute of Technology, S-100 44 Stockholm, Sweden Internet: `ulfj@math.kth.se`
C. Lemaréchal	INRIA Rhone-Alpes, ZIRST - 655 avenue de l'Europe 38330 Montbonnot Saint-Martin, France Internet: `Claude.Lemarechal@inria.fr`
M. Mesbahi	Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109 Internet: `mesbahi@hafez.jpl.nasa.gov`
M. V. Nayakkankuppam	Department of Mathematics and Statistics, University of Maryland, Baltimore County, Baltimore, MD 21250 Internet: `madhu@cs.nyu.edu`
S.-I. Niculescu	HEUDIASYC (UMR CNRS 6599), UTC-Compiègne, BP 20529, 60205, Compiègne, Cédex, France Internet: `silviu@hds.utc.fr`
J. Oishi	Production Engineering Department Laboratory, NEC Corporation, Tomagawa Plant 1753 Shimonumabe, Nakahara-ku Kawasaki, 211 Japan Internet: `bigstone@msa.biglobe.ne.jp`
F. Oustry	INRIA Rhone-Alpes, ZIRST - 655 avenue de l'Europe 38330 Montbonnot Saint-Martin, France Internet: `Francois.Oustry@inria.fr`
M. L. Overton	Computer Science Department, Courant Institute, New York University, New York, NY 10012 Internet: `overton@cs.nyu.edu`

F. Paganini	Department of Electrical Engineering, UCLA, Los Angeles, CA 90095 Internet: `paganini@ee.ucla.edu`
G. P. Papavassilopoulos	Department of EE—Systems, University of Southern California, Los Angeles, CA 90089 Internet: `yorgos@bode.usc.edu`
A. Rantzer	Lund Institute of Technology, Box 118, s-221 00 Lund, Sweden Internet: `rantzer@control.lth.se`
M. G. Safonov	Department of EE—Systems, University of Southern California, Los Angeles, CA 90089 Internet: `msafanov@usc.edu`
C. W. Scherer	Mechanical Engineering Systems and Control Group, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands Internet: `c.w.scherer@wbmt.tudelft.nl`
A. Trofino	Department of Automation and Systems, Universidade Federal de Santa Catarina, 88040-900 Florianópolis, SC, Brazil Internet: `trofino@lcmi.ufsc.br`
S.-P. Wu	Numerical Technology, Inc., Santa Clara, CA 95051
K. Y. Yang	Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, MA 02420 Internet: `kyang@ll.mit.edu`

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Contents

Preface xvii

Notation xxv

I Introduction 1

1 Robust Decision Problems in Engineering: A Linear Matrix Inequality Approach 3

L. El Ghaoui and S.-I. Niculescu

1.1 Introduction . . . 3
1.1.1 Basic idea . . . 3
1.1.2 Approximation and complexity . . . 4
1.1.3 Purpose of the chapter . . . 4
1.1.4 Outline . . . 4
1.2 Decision problems with uncertain data . . . 5
1.2.1 Decision problems . . . 5
1.2.2 Using uncertainty models . . . 5
1.2.3 Robust decision problems . . . 6
1.2.4 Lagrange relaxations for min-max problems . . . 7
1.3 Uncertainty models . . . 8
1.3.1 Linear-fractional models . . . 9
1.3.2 Operator point of view . . . 11
1.3.3 Examples . . . 12
1.3.4 Toward other models . . . 13
1.4 Robust SDP . . . 14
1.4.1 Definition . . . 14
1.4.2 Lagrange relaxation of a robust SDP . . . 15
1.4.3 A dual view: Rank relaxation . . . 16
1.4.4 Extension to nonconvex nominal problems . . . 17
1.4.5 On quality of relaxations . . . 18
1.4.6 Link with Lyapunov functions . . . 18
1.5 Algorithms and software for SDP . . . 21
1.5.1 Facts about SDP and related problems . . . 21
1.5.2 Interior-point methods . . . 22
1.5.3 Nondifferentiable optimization methods . . . 24
1.5.4 Software . . . 24
1.6 Illustrations in control . . . 25
1.6.1 Optimal control and SDP duality . . . 25

1.6.2 Impulse differential systems 26
1.6.3 Delay differential systems 27
1.6.4 Open-loop and predictive robust control 28
1.7 Robustness out of control 29
1.7.1 LMIs in combinatorial optimization 29
1.7.2 Interval calculus 30
1.7.3 Structured linear algebra 31
1.7.4 Robust portfolio optimization 33
1.8 Perspectives and challenges 33
1.8.1 Algorithms and software 33
1.8.2 Nonconvex LMI-constrained problems 34
1.8.3 Dynamical systems over cones 35
1.8.4 Quality of relaxations for uncertain systems 36
1.9 Concluding remarks 37

II Algorithms and Software 39

2 Mixed Semidefinite–Quadratic–Linear Programs 41
Jean-Pierre A. Haeberly, Madhu V. Nayakkankuppam, and Michael L. Overton
2.1 Introduction 41
2.2 A primal-dual interior-point method 44
2.3 Software 47
2.3.1 Some implementation issues 47
2.3.2 Using SDPpack 48
2.3.3 Miscellaneous features 50
2.4 Applications 51
2.5 Numerical results 51
2.6 Other computational issues 53
2.7 Conclusions 54

3 Nonsmooth Algorithms to Solve Semidefinite Programs 57
Claude Lemaréchal and François Oustry
3.1 Introduction 57
3.2 Classical methods 60
3.3 A short description of standard bundle methods 61
3.4 Specialized bundle methods 64
3.4.1 Rich oracle: Dual methods 64
3.4.2 Rich oracle: A primal approach 65
3.4.3 A large class of bundle methods 67
3.5 Second-order schemes 70
3.5.1 Local methods 70
3.5.2 Decomposition of the space 70
3.5.3 Second-order developments 71
3.5.4 Quadratic step 71
3.5.5 The dual metric 72
3.5.6 A second-order bundle method 73
3.6 Numerical results 74
3.6.1 Influence of ε 74
3.6.2 Large-scale problems 75

3.6.3 Superlinear convergence . . . 76
3.7 Perspectives . . . 76

4 sdpsol: A Parser/Solver for Semidefinite Programs with Matrix Structure 79
Shao-Po Wu and Stephen Boyd
4.1 Introduction . . . 79
4.1.1 Max-det problems and SDPs . . . 79
4.1.2 Matrix structure . . . 80
4.1.3 Implications of the matrix structure . . . 81
4.1.4 sdpsol and related work . . . 81
4.2 Language design . . . 82
4.2.1 Matrix variables and affine expressions . . . 82
4.2.2 Constraints and objective . . . 83
4.2.3 Duality . . . 83
4.3 Implementation . . . 84
4.3.1 Handling equality constraints . . . 85
4.4 Examples . . . 86
4.4.1 Lyapunov inequality . . . 86
4.4.2 Popov analysis . . . 86
4.4.3 $\mathcal{D}$-optimal experiment design . . . 88
4.4.4 Lyapunov exponent analysis . . . 90
4.5 Summary . . . 91

III Analysis 93

5 Parametric Lyapunov Functions for Uncertain Systems: The Multiplier Approach 95
Minyue Fu and Soura Dasgupta
5.1 Introduction . . . 95
5.2 Multipliers and robust stability . . . 97
5.3 Parametric KYP lemma . . . 98
5.4 Main results on parametric Lyapunov functions . . . 100
5.5 Comparison with other schemes . . . 103
5.5.1 Generalized Popov criterion . . . 103
5.5.2 The AQS test . . . 105
5.6 Generalization to discrete-time systems . . . 106
5.7 Conclusions . . . 107

6 Optimization of Integral Quadratic Constraints 109
Ulf Jönsson and Anders Rantzer
6.1 Introduction . . . 109
6.2 IQC-based robustness analysis . . . 111
6.2.1 Robust stability analysis based on IQCs . . . 112
6.2.2 Robust performance analysis based on IQCs . . . 113
6.3 Signal specifications . . . 115
6.4 A robust performance example . . . 116
6.5 LMI formulations of robustness tests . . . 117
6.6 Numerical examples . . . 121
6.7 Conclusions . . . 125

6.8 Appendix . . . 126
6.8.1 Proof of Theorem 6.2 and Remark 6.4 . . . 126
6.8.2 Multiplier sets for Example 6.3 . . . 127

7 Linear Matrix Inequality Methods for Robust $\mathbf{H}_2$ Analysis: A Survey with Comparisons 129
Fernando Paganini and Eric Feron
7.1 Introduction . . . 129
7.2 Problem formulation . . . 130
7.2.1 The $\mathbf{H}_2$ norm . . . 130
7.2.2 Robust $\mathbf{H}_2$-performance . . . 131
7.3 State-space bounds involving causality . . . 133
7.3.1 The impulse response interpretation . . . 133
7.3.2 Stochastic white noise interpretation . . . 135
7.3.3 Refinements for LTI uncertainty . . . 137
7.3.4 A dual state-space bound . . . 137
7.4 Frequency domain methods and their interpretation . . . 139
7.4.1 The frequency domain bound and LTI uncertainty . . . 139
7.4.2 Set descriptions of white noise and losslessness results . . . 141
7.4.3 Computational aspects . . . 143
7.5 Discussion and comparisons . . . 143
7.5.1 NLTV uncertainty . . . 143
7.5.2 LTI uncertainty . . . 145
7.5.3 A common limitation: Multivariable noise . . . 146
7.6 Numerical examples . . . 147
7.6.1 Comparing $\mathbf{J}_s$ and $\mathbf{J}_s'$ . . . 147
7.6.2 NLTV case: Gap between stochastic and worst-case white noise . . 148
7.6.3 LTI case: Tighter bounds due to causality . . . 148
7.6.4 LTI case: Tighter bounds due to frequency-dependent multipliers . 148
7.6.5 LTI case: Using both causality and dynamic multipliers . . . 148
7.6.6 A large-scale engineering example . . . 149
7.7 Conclusion . . . 150

IV Synthesis 153

8 Robust $\mathbf{H}_2$ Control 155
Kyle Y. Yang, Steven R. Hall, and Eric Feron
8.1 Introduction . . . 155
8.2 Notation . . . 157
8.3 Analysis problem statement and approach . . . 158
8.4 Upper bound computation via families of linear multipliers . . . 160
8.4.1 Choosing the right multipliers . . . 160
8.4.2 Computing upper bounds on robust $\mathbf{H}_2$-performance . . . 160
8.5 Robust $\mathbf{H}_2$ synthesis . . . 164
8.5.1 Synthesis problem statement . . . 164
8.5.2 Synthesis via LMIs . . . 165
8.5.3 Synthesis via a separation principle . . . 166
8.6 Implementation of a practical design scheme . . . 167
8.6.1 Analysis step . . . 168
8.6.2 Synthesis step . . . 168

8.6.3 Design examples 169
8.7 Summary 173

9 A Linear Matrix Inequality Approach to the Design of Robust $\mathbf{H}_2$ Filters 175
Carlos E. de Souza and Alexandre Trofino
9.1 Introduction 175
9.2 The filtering problem 176
9.3 Full-order robust filter design 177
9.4 Reduced-order robust filter design 183
9.5 Conclusions 185

10 Robust Mixed Control and Linear Parameter-Varying Control with Full Block Scalings 187
Carsten W. Scherer
10.1 Introduction 187
10.2 System description 188
10.3 Robust performance analysis with constant Lyapunov matrices 189
10.3.1 Well-posedness and robust stability 189
10.3.2 Robust quadratic performance 191
10.3.3 Robust $\mathbf{H}_2$ performance 193
10.3.4 Robust generalized $\mathbf{H}_2$ performance 194
10.3.5 Robust bound on peak-to-peak gain 195
10.4 Dualization 196
10.5 How to verify the robust performance tests 197
10.6 Mixed robust controller design 199
10.6.1 Synthesis inequalities for output feedback control 199
10.6.2 Synthesis inequalities for state-feedback control 201
10.6.3 Elimination of controller parameters 201
10.7 LPV design with full scalings 202
10.8 Conclusions 206
10.9 Appendix: Auxiliary results on quadratic forms 206
10.9.1 A full block $\mathcal{S}$-procedure 206
10.9.2 Dualization 206
10.9.3 Solvability test for a quadratic inequality 207

11 Advanced Gain-Scheduling Techniques for Uncertain Systems 209
Pierre Apkarian and Richard J. Adams
11.1 Introduction 209
11.2 Output-feedback synthesis with guaranteed L_2-gain performance 210
11.2.1 Extensions to multiobjective problems 214
11.3 Practical validity of gain-scheduled controllers 214
11.4 Reduction to finite-dimensional problems 215
11.4.1 Overpassing the gridding phase 218
11.5 Reducing conservatism by scaling 218
11.6 Control of a two-link flexible manipulator 221
11.6.1 Problem description 221
11.6.2 Numerical examples 224
11.6.3 Frequency and time-domain validations 226
11.7 Conclusions 228

12 Control Synthesis for Well-Posedness of Feedback Systems **229**
Tetsuya Iwasaki
12.1 Introduction 229
12.2 Well-posedness analysis 230
12.2.1 Exact condition 230
12.2.2 P-separator 231
12.3 Synthesis for well-posedness 232
12.3.1 Problem formulation 232
12.3.2 Main results 235
12.3.3 Proof of the main results 237
12.4 Applications to control problems 240
12.4.1 Fixed-order stabilization 240
12.4.2 Robust control 242
12.4.3 Gain-scheduled control 244
12.5 Concluding remarks 246

V Nonconvex Problems 249

13 Alternating Projection Algorithms for Linear Matrix Inequalities Problems with Rank Constraints **251**
Karolos M. Grigoriadis and Eric B. Beran
13.1 Introduction 251
13.2 LMI-based control design with rank constraints 252
13.2.1 Control problem formulation 252
13.2.2 Stabilization with prescribed degree of stability 253
13.2.3 $\mathbf{H}_\infty$ control 255
13.3 Alternating projecting schemes 257
13.3.1 The standard alternating projection method 257
13.3.2 The directional alternating projection method 258
13.4 Mixed alternating projection/SP (AP/SP) design for low-order control . . 259
13.5 Numerical experiments 263
13.5.1 Randomly generated stabilizable systems 263
13.5.2 Helicopter example 264
13.5.3 Spring-mass systems 265
13.6 Conclusions 267

14 Bilinearity and Complementarity in Robust Control **269**
Mehran Mesbahi, Michael G. Safonov, and George P. Papavassilopoulos
14.1 Introduction 269
14.1.1 Preliminaries on convex analysis 270
14.1.2 Background on control theory 271
14.2 Methodological aspects of the BMI 274
14.2.1 Limitations of the LMI approach 275
14.2.2 Proof of Theorem 14.1 276
14.2.3 Why is the BMI formulation important? 278
14.3 Structural aspects of the BMI 279
14.3.1 Concave programming approach 279
14.3.2 Cone programming approach 282
14.4 Computational aspects of the BMI 289
14.5 Conclusion 291

VI Applications 293

15 Linear Controller Design for the NEC Laser Bonder via Linear Matrix Inequality Optimization 295
Junichiro Oishi and Venkataramanan Balakrishnan
15.1 Introduction 295
15.2 Controller design using the Youla parametrization and LMIs 296
15.2.1 Youla parametrization 297
15.2.2 Controller design specifications as LMI constraints 299
15.3 Design of an LTI controller for the NEC Laser Bonder 301
15.3.1 Modeling of NEC Laser Bonder 301
15.3.2 Design specifications 303
15.3.3 Setting up the LMI problem 304
15.3.4 Numerical results 305
15.4 Conclusions 306

16 Multiobjective Robust Control Toolbox for Linear-Matrix-Inequality-Based Control 309
Stéphane Dussy
16.1 Introduction 309
16.2 Theoretical framework 310
16.2.1 Specifications 310
16.2.2 LFR of the open-loop system 311
16.2.3 LFR of the controller 311
16.2.4 LMI conditions 311
16.2.5 Main motivations for the toolbox 313
16.3 Toolbox contents 314
16.3.1 General structure of the toolbox 314
16.3.2 Description of the driver `mrct` 314
16.3.3 Description of the main specific functions 315
16.4 Numerical examples 316
16.4.1 An inverted pendulum benchmark 316
16.4.2 The rotational/translational proof mass actuator 318
16.5 Concluding remarks 320

17 Multiobjective Control for Robot Telemanipulators 321
Jean Pierre Folcher and Claude Andriot
17.1 Introduction 321
17.2 Control of robot telemanipulators 322
17.2.1 Teleoperation systems 322
17.2.2 Background on network theory 322
17.2.3 Toward an ideal scaled teleoperator 324
17.3 Multiobjective robust control 326
17.3.1 Problem statement 326
17.3.2 Control objectives 326
17.3.3 Conditions for robust synthesis 328
17.3.4 Reconstruction of controller parameters 330
17.3.5 Checking the synthesis conditions 331
17.4 Force control of manipulator RD500 332
17.4.1 Single-joint slave manipulator model 332
17.4.2 Controller design 334

17.4.3 Results 336
17.5 Conclusion and perspectives 338

Bibliography **341**

Index **369**

Preface

Linear matrix inequalities (LMIs) have emerged recently as a useful tool for solving a number of control problems. The basic idea of the LMI method in control is to interpret a given control problem as a *semidefinite programming* (SDP) problem, i.e., an optimization problem with linear objective and positive semidefinite constraints involving symmetric matrices that are affine in the decision variables.

The LMI formalism is relevant for many reasons. First, writing a given problem in this form brings an efficient, numerical solution. Also, the approach is particularly suited to problems with "uncertain" data and multiple (possibly conflicting) specifications. Finally, this approach seems to be widely applicable, not only in control, but also in other areas where uncertainty arises.

Purpose and intended audience

Since the early 1990s, with the developement of *interior-point methods* for solving SDP problems, the LMI approach has witnessed considerable attention in the control area (see the regularity of the invited sessions in the control conferences and workshops). Up to now, two self-contained books related to this subject have appeared. The book *Interior Point Polynomial Methods in Convex Programming: Theory and Applications,* by Nesterov and Nemirovskii, revolutionarized the field of optimization by showing that a large class of nonlinear convex programming problems (including SDP) can be solved very efficiently. A second book, also published by SIAM in 1994, *Linear Matrix Inequalities in System and Control Theory,* by Boyd, El Ghaoui, Feron, and Balakrishnan, shows that the advances in convex optimization can be successfully applied to a wide variety of difficult control problems.

At this point, a natural question arises: Why another book on LMIs?

One aim of this book is to describe, for the researcher in the control area, several important advances made both in algorithms and software and in the important issues in LMI control pertaining to analysis, design, and applications. Another aim is to identify several important issues, both in control and optimization, that need to be addressed in the future.

We feel that these challenging issues require an interdisciplinary research effort, which we sought to foster. For example, Chapter 1 uses an optimization formalism, in the hope of encouraging researchers in optimization to look at some of the important ideas in LMI control (e.g., deterministic uncertainty, robustness) and seek nonclassical applications and challenges in the control area. Bridges go both ways, of course: for example, the "primal-dual" point of view that is so successful in optimization is also important in control.

Book outline

In our chapter classification, we sought to provide a continuum from numerical methods to applications, via some theoretical problems involving analysis and synthesis for uncertain systems. Basic notation and acronyms are listed after the preface.

After this outline, we provide some alternative keys for reading this book.

Part I: Introduction

Chapter 1: Robust Decision Problems in Engineering: A Linear Matrix Inequality Approach, by L. El Ghaoui and S.-I. Niculescu.

This chapter is an introduction to the "LMI method" in robust control theory and related areas. A large class of engineering problems with uncertainty can be expressed as optimization problems, where the objective and constraints are perturbed by unknown-but-bounded parameters. The authors present a robust decision framework for a large class of such problems, for which (approximate) solutions are computed by solving optimization problems with LMI constraints.

The authors emphasize the wide scope of the method, and the anticipated interplay between the tools developed in LMI robust control, and related areas where uncertain decision problems arise.

Part II: Algorithms and software

Chapter 2: Mixed Semidefinite–Quadratic–Linear Programs, by J.-P. A. Haeberly, M. V. Nayakkankuppam, and M. L. Overton.

The authors consider mixed semidefinite–quadratic–linear programs. These are linear optimization problems with three kinds of cone constraints, namely, the semidefinite cone, the quadratic cone, and the nonnegative orthant. The chapter outlines a primal-dual path-following method to solve these problems and highlights the main features of SDPpack, a Matlab package that solves such programs. Furthermore, the authors give some examples where such mixed programs arise and provide numerical results on benchmark problems.

Chapter 3: Nonsmooth Algorithms to Solve Semidefinite Programs, by C. Lemaréchal and F. Oustry.

Today, SDP problems are usually solved by interior-point methods, which are elegant, efficient, and well suited. However, they have limitations, particularly in large-scale or ill-conditioned cases. On the other hand, SDP is an instance of nonsmooth optimization (NSO), which enjoys some particular structure. This chapter briefly reviews the works that have been devoted to solving SDP with NSO tools and presents some recent results on bundle methods for SDP. Finally, the authors outline some possibilities for future work.

Chapter 4: `sdpsol`: A Parser/Solver for Semidefinite Programs with Matrix Structure, by S.-P. Wu and S. Boyd.

This chapter describes a parser/solver for a class of LMI problems, the so-called max-det problems, which arise in a wide variety of engineering problems. These problems often have matrix structure, which has two important practical ramifications: first, it makes the job of translating the problem into a standard SDP or max-det format tedious, and, second, it opens the possibility of exploiting the structure to speed up the computation.

In this chapter the authors describe the design and implementation of `sdpsol`, a parser/solver for SDPs and max-det problems. `sdpsol` allows problems with matrix structure to be described in a simple, natural, and convenient way.

Part III: Analysis

Chapter 5: Parametric Lyapunov Functions for Uncertain Systems: The Multiplier Approach, by M. Fu and S. Dasgupta.

This chapter proposes a *parametric multiplier approach* to deriving parametric Lyapunov functions for robust stability analysis of linear systems involving uncertain parameters. This new approach generalizes the traditional multiplier approach used in the absolute stability literature where the multiplier is independent of the uncertain parameters. The main result provides a general framework for studying *multiaffine Lyapunov functions*. It is shown that these Lyapunov functions can be found using LMI techniques. Several known results on parametric Lyapunov functions are shown to be special cases.

Chapter 6: Optimization of Integral Quadratic Constraints, by U. Jönsson and A. Rantzer.

A large number of performance criteria for uncertain and nonlinear systems can be unified in terms of integral quadratic constraints. This makes it possible to systematically combine and evaluate such criteria in terms of LMI optimization.

A given combination of nonlinear and uncertain components usually satisfies infinitely many integral quadratic constraints. The problem to identify the most appropriate constraint for a given analysis problem is convex but infinite dimensional. A systematic approach, based on finite-dimensional LMI optimization, is suggested in this chapter. Numerical examples are included.

Chapter 7: Linear Matrix Inequality Methods for Robust $\mathbf{H}_2$ Analysis: A Survey with Comparisons, by F. Paganini and E. Feron.

This chapter provides a survey of different approaches for the evaluation of $\mathbf{H}_2$-performance in the worst case over structured system uncertainty, all of which rely on LMI computation. These methods apply to various categories of parametric or dynamic uncertainty (linear time-invariant (LTI), linear time-varying (LTV), or nonlinear time-varying (NLTV)) and build on different interpretations of the $\mathbf{H}_2$ criterion. It is shown nevertheless how they can be related by using the language of LMIs and the so-called $\mathcal{S}$-procedure for quadratic signal constraints. Mathematical comparisons and examples are provided to illustrate the relative merits of these approaches as well as a common limitation.

Part IV: Synthesis

Chapter 8: Robust $\mathbf{H}_2$ Control, by K. Y. Yang, S. R. Hall, and E. Feron.

In this chapter, the problem of analyzing and synthesizing controllers that optimize the $\mathbf{H}_2$ performance of a system subject to LTI uncertainties is considered. A set of upper bounds on the system performance is derived, based on the theory of stability multipliers and the solution of an original optimal control problem. A Gauss–Seidel-like algorithm is proposed to design robust and efficient controllers via LMIs. An efficient solution procedure involves the iterative solution of Riccati equations both for analysis and synthesis purposes. The procedure is used to build robust and efficient controllers for a space-borne active structural control testbed, the Middeck Active Control Experiment (MACE). Controller design cycles are now short enough for experimental investigation.

Chapter 9: A Linear Matrix Inequality Approach to the Design of Robust $\mathbf{H}_2$ Filters, by C. E. de Souza and A. Trofino.

This chapter is concerned with the robust minimum variance filtering problem for linear continuous-time systems with parameter uncertainty in all the matrices of the system state-space model, including the coefficient matrices of the noise signals. The