Functions of a Complex Variable

Theory and Technique

George F. Carrier
Max Krook
Carl E. Pearson

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49

Functions of a Complex Variable

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Theory and Technique

George F. Carrier
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contents

4 *conformal mapping* 111

5 *special functions* 183

6 *asymptotic methods* 241

7 *transform methods* 301

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preface

In addition to being a rewarding branch of mathematics in its own right, the theory of functions of a complex variable underlies a large number of enormously powerful techniques which find their application not only in other branches of mathematics but also in the sciences and in engineering. Chapters 1, 2, and 5 of this book provide concisely but honestly the classical aspects of the theory of functions of a complex variable; the rest of the book is devoted to a detailed account of various techniques and the ideas from which they evolve. Many of the illustrative examples are phrased in terms of the physical contexts in which they might arise; however, we have tried to be consistent in including a mathematical statement of each problem to be discussed.

For the acquisition of skill in the use of these techniques, practice is even more important than instruction. Accordingly, we have inserted many exercises, including some which are essential parts of the text. The reader who fails to carry out a substantial number of these exercises will have missed much of the value of this book.

The individual chapters segregate specific topical items, but many readers will find it profitable to study selected parts of Chaps. 3 to 7 as they encounter the related underlying theory in Chap. 2.

We hope that through our presentation the reader will be able to discern the fascination of complex-function theory, recognize its power, and acquire skill in its use.

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erratum

On page 340, Exercise 8 should read as follows:
8.(a) Let $g(x, y, z, t; \xi, \eta, \zeta, \tau)$ satisfy the wave equation

$$g_{xx} + g_{yy} + g_{zz} - \frac{1}{c^2} g_{tt} = \delta(x - \xi)\delta(y - \eta)\delta(z - \zeta)\delta(t - \tau).$$

That is, in two places, the symbol ξ should be replaced by ζ.

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1

complex numbers and their elementary properties

1-1 Origin and Definition

Complex numbers originated from the desire for a symbolic representation for the solution of such equations as $x^2 + 1 = 0$. Continued usage for this purpose gradually endowed them with some degree of conceptual existence; however, they were not generally accepted as providing a legitimate extension of real numbers until experience showed that their use gave a completeness and insight that had previously been lacking. It was found, for example, that if complex numbers were allowed, then every polynomial had a zero; moreover, such results as the previously mysterious divergence at $x = 1$ of the power series for $(1 + x^2)^{-1}$ became more explicable.

Complex numbers are now so widely used in applied mathematics (for example, in evaluation of integrals, series representations of functions, and solutions of ordinary and partial differential equations) that they tend to be accepted without question. It was not always so; in forcing their way into mathematics, these "imaginary" numbers experienced as much opposition as negative numbers had in their earlier