APPLIED OPTIMIZATION

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ONLINE STORAGE SYSTEMS AND TRANSPORTATION PROBLEMS WITH APPLICATIONS

Optimization Models and Mathematical Solutions

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Optimization Models and Mathematical Solutions

by

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Created in the United States of America

to my parents

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Preface

This book covers the analysis and development of online algorithms involving exact optimization and heuristic techniques, and their application to solve two real life problems.

The first problem is concerned with a complex technical system: a special carousel based high-speed storage system - Rotastore. It is shown that this logistic problem leads to an NP-hard *Batch PreSorting Problem* (BPSP) which is not easy to solve optimally in offline situations. We consider a polynomial case and develope an exact algorithm for offline situations. Competitive analysis showed that the proposed online algorithm is 3/2-competitive. Online algorithms with lookahead improve the online solutions in particular cases. If the capacity constraint on additional storage is neglected the problem has a totally unimodular polyhedron.

The second problem originates in the health sector and leads to a vehicle routing problem. We demonstrate that reasonable solutions for the offline case covering a whole day with a few hundred orders can be constructed with a heuristic approach, as well as by simulated annealing. Optimal solutions for typical online instances are computed by an efficient column enumeration approach leading to a set partitioning problem and a set of routing-scheduling subproblems. The latter are solved exactly with a branch-and-bound method which prunes nodes if they are value-dominated by previous found solutions or if they are infeasible with respect to the capacity or temporal constraints. Our branch-and-bound method is suitable to solve any kind of sequencing-scheduling problem involving accumulative objective functions and constraints, which can be evaluated sequentially. The column enumeration approach developed to solve this hospital problem is of general nature and thus can be embedded into any decision-support system involving assigning, sequencing and scheduling.

The book is aimed at practioners and scientists in operation research especially those interested in online optimization. The target audience are readers interested in fast solutions of batch presorting and vehicle routing problems or software companies producing decision support systems. Students and graduates in mathematics, physics, operations research, and businesses with interest in modeling and solving real optimization problems will also benefit from this book and can experience how online optimization enters into real world problems.

Structure of this Book

This book is organized as follows. Chapter 2 addresses the BPSP, where a formal definition of the BPSP is introduced (Section 2.1) and several modeling approaches are proposed (see Section 2.2). Complexity issues of some formulations are investigated in Section 2.3 and Section 2.4. For one polynomial case of the BPSP several algorithms are presented and compared in Section 2.5. In Chapter 3 we consider a concrete application of the BPSP - carousel based storage system Rotastore. In Section 3.1 we describe the system performance, and in Section 3.2 the numerical results of the experiments are presented.

Chapter 4 focuses on the *Vehicle Routing problem with Pickup and Delivery and Time Windows* (VRPPDTW), adapted for hospital transportation problems. After introducing some notations (Subsection 4.2.1), we suggest several approaches we have developed to solve this problem, including a MILP formulation (Subsection 4.3.1), a branch-and-bound approach (Subsection 4.3.2), a column enumeration approach (Subsection 4.3.3), and heuristic methods (Section 4.4). In Chapter 5 we describe a problem related to a hospital project with the University Hospital in Homburg. Detailed numerical results for our solution approaches related to the VRPPDTW are collected in Section 5.2.

Conventions and Abbreviations

The following table contains in alphabetic order the abbreviations used in this book.

Abbreviation	*Meaning*
B&B	Branch-and-Bound
B&C	Branch-and-Cut
BPSP	Batch PreSorting Problem
CEA	column enumeration approach
IP	Integer Programming
LP	Linear Programming
MCP	Mixed Complementarity Problem
MILP	Mixed Integer Linear Programming
MINLP	Mixed Integer Nonlinear Programming
RH	reassignment heuristic
SA	simulated annealing
SAT	satisfiability problem
SH	sequencing heuristic
s.t.	subject to
TS	tabu search
VNS	variable neighborhood search
VRP	Vehicle Routing Problem
VRPPDTW	VRP with Pickup and Delivery and Time Windows
w.r.t.	with respect to

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Chapter 1

INTRODUCTION

What do a logistics manager responsible for an inventory storage system and a vehicle fleet dispatcher in a hospital campus have in common? They both have to consider new objects arriving at short notice and to decide on what to do with them, how to assign them to given resources or how to modify previously made decisions. This means they both need to make decisions based on data suffering from incomplete knowledge about near future events. Online optimization is a discipline in mathematical optimization and operations research which provides the mathematical framework and algorithms for dealing appropriately with such situations.

1.1. Optimization Everywhere

The need for applying optimization arises in many areas: finance, space industry, biosystems, textile industry, mineral oil, process and metal industry, and airlines to name a few. Mathematical programming is a very natural and powerful way to solve problems appearing in these areas. In particular, see [12], [18], [23], [37] and [83] for application examples. One might argue that low structure systems can probably be handled well without optimization. However, for the analysis and development of real life complex systems (that have many degrees of freedom, underlying numerous restrictions *etc.*) the application of optimization techniques is unavoidable. It would not be an exaggeration even to say that any decision problem is an optimization problem. Despite their diversity real world optimization problems often share many common features, *e.g.,* they have similar mathematical kernels such as flow, assignment or knapsack structures.

One further common feature of many real life decision problems is the online nature aspect, *i.e.,* decision making is based on partial, insufficient information or without any knowledge of the future. One approach (not treated in this book) to solve problems with only partial or insufficient information is optimization under uncertainty (*cf.* [45], [50], or [88]), and especially, stochastic programming (*cf.* [14], [53], [77], or [78]). In that case, the problem is still solved as an offline problem.

However, it is not always appropriate to solve a problem offline. If we cannot make any assumptions on future data, only the currently available data can be used. In such situations online optimization is recommended. We can list a number of problems that were originally formulated as offline problems but which in many practical applications are used in their online versions: the bin packing problem, the list update problem, the k-server problem, the vehicle routing problem, and the pickup and delivery problem to name a few.

Special optimization techniques for online applications exploit the online nature of the decision process. Usually, a sequence of online optimization problems is solved when advancing in time and more data become available. Therefore, online optimization can be much faster than offline optimization (which uses the complete input data). To estimate the quality of a sequence of solutions obtained by online optimization one can only compare it with the overall solution produced by an offline algorithm afterwards. A powerful technique to estimate the performance of online algorithms is the competitive analysis (*cf.* [11]). A good survey on online optimization and competitive analysis can be found in [4], [11], [30]. Online optimization and competitive analysis are based on generic principles and can be beneficial in completely different areas such as the storage system and transportation problem considered in this book.

At first we consider an example of a complex technical system, namely a special carousel based high-speed storage system - *Rotastore* [73], which not only allows storing ([56], [57]) but also performs sorting ([49], [70]). Sorting actions and assignment to storing locations are fulfilled in real time, but the information horizon may be rather narrow. The quality of the corresponding decisions strongly influences the performance of the system in general; thus the need to improve the quality of the decisions. Due to the limited information horizon online optimization is a promising approach to solve these problems.

In our second case study, the conditions for the decision making process in hospital transportation are similar: the orders often are not known in advance, the transportation network may be changed dynamically. The efficiency of order assignment and scheduling of the transport system can influence the operation of the whole hospital. That assumes, in this case, not only economical aspects, but, at first of all, human health and life issues.

As will be shown in this book, the mathematical base for the first problem is the *Batch PreSorting Problem* (BPSP), for the second one we naturally can use an online variant of the *Vehicle Routing problem with Pickup and Delivery and Time Windows* (VRPPDTW). The efficient application of the corresponding solution methods allows to improve the performance of both systems compared to the current real life situation.

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Chapter 2

BATCH PRESORTING PROBLEMS. I MODELS AND SOLUTION APPROACHES

This chapter is organized as follows: at first, we describe the problem and give a short classification. In Section 2.2 different formulations of the BPSP are presented. In Subsection 2.2.2 we consider an optimization version of $BPSP_1$. In Subsection 2.2.3 we formulate $BPSP_2$ and $BPSP_3$ as decision problems and additionally introduce optimization models. The complexity status of $BPSP_2$ is investigated in Section 2.3, and in Section 2.4 we show that there is a polynomial version of the BPSP. Also we consider a special subcase of a BPSP with $N^L = 2$ in offline and online situations and present corresponding algorithms in Section 2.5. Finally, in Section 2.6, some results derived for BPSPs with $N^L = 2$ are adapted to general BPSP.

2.1. Problem Description and Classification

We consider the problem of finding a finite sequence of objects of different types, that guarantees an optimal assignment of objects to given physical storage layers with a pre-sorting facility of limited capacity. This problem will be called the *Batch PreSorting Problem* (BPSP), because the objects have to be sorted within one batch before they are assigned to the layers. After sorting, the object with number i will be assigned to layer i. For a more transparent presentation we speak of colors instead of types and thus consider all objects of type k as having the same color k. We present three types of BPSP with different objective functions. The