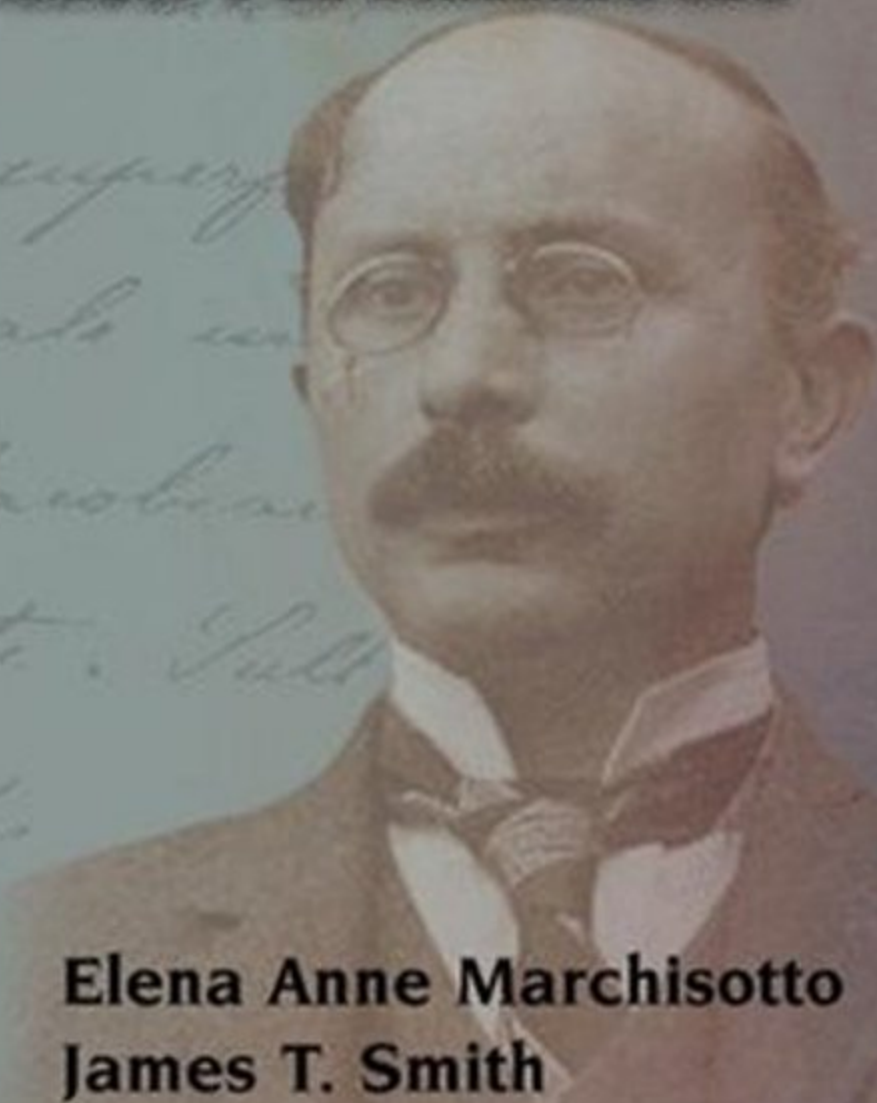


The Legacy of Mario Pieri in Geometry and Arithmetic



**Elena Anne Marchisotto
James T. Smith**



Mario Pieri

Mario Pieri

*In loving memory of Helen Cullura Corie,
with gratitude for her inspiration and support*

Elena Anne Marchisotto
James T. Smith

The Legacy of Mario Pieri
in Geometry and Arithmetic

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Foreword

by Ivor Grattan-Guinness

One of the distortions in most kinds of history is an imbalance between the study devoted to major figures and to lesser ones, concerning both achievements and influence: the Great Ones may be studied to death while the others are overly ignored and thereby remain underrated. In my own work in the history of mathematics I have noted at least a score of outstanding candidates for neglect, of whom Mario Pieri (1860–1913) is one. A most able contributor to geometry, arithmetic and mathematical analysis, and mathematical logic during his rather short life, his work and its legacy are not well known. The main reason is that Pieri worked “in the shadow of giants,” to quote one of the authors of this volume.¹

Born into a scholarly family in Lucca, Pieri was educated briefly at the University of Bologna and principally at the prestigious Scuola Normale Superiore, in Pisa; under the influence of Luigi Bianchi (1856–1928) he wrote there his doctoral dissertations on algebraic and differential geometry. During his twenties came appointments in Turin, first at the military academy and then also at the university, where he fell under the sway of Corrado Segre (1863–1924) in algebraic geometry, and Giuseppe Peano (1858–1932) in the foundations of arithmetic, mathematical analysis, and mathematical logic. From 1900 to 1908 he held a chair at the University of Catania before moving to Parma, where he died from cancer. During these last years Pieri continued and broadened his interests, especially in other parts of geometry and in related topics such as vector analysis.

As part of his contributions to geometry and logic Pieri took up the growing interest of that time in the axiomatisation of mathematical theories and the attendant reduction in the number of primitive notions. Among other noteworthy features, he pioneered talk of the “hypothetico-deductive” method in mathematics. David Hilbert (1862–1943) had been emphasising this method from the mid-1890s onwards, especially in geometry; and Pieri’s axiomatisation of geometry (especially its projective part) may equal in calibre Hilbert’s own axiom system. In addition, at that time Pieri’s work was noticed and praised in 1903 by another student of foundations, Bertrand Russell (1872–1970). However, when the University of Kazan awarded its Lobachevsky Prize that year for recent contributions to geometry, Hilbert won, with Pieri receiving an honourable mention. After Pieri’s death in 1913, all of his work gradually disappeared from attention, until from the mid-1920s onwards some of his work on geometry gained the admiration of another giant, Alfred Tarski (1902–1983).

¹ Marchisotto 1995.

Pieri has also not been completely ignored historically; there was a photoreprint edition of his writings on foundational matters, and an edition of letters addressed to him.² Now the authors of this book and its two planned successor volumes have not only studied his life and work and their historical context in great detail but have also translated into English several of his principal papers. Thereby they exhibit his proper place in the histories of mathematics and of logic, and also clarify the context of some surrounding Great Ones. Their efforts supplement the recent revival of work on Peano himself that Clara Silvia Roero has led.³ One looks forward to further attention paid to other major “Peanists,” especially Cesaro Burali-Forti (1861–1931), who was a fellow student in Pisa, and Alessandro Padoa (1868–1937). But in the meantime my list of neglected figures requiring special consideration has definitely decreased by one.

² Pieri 1980; Arrighi 1997.

³ Roero 2001; Roero, Nervo, and Armano 2002; Peano 2002; Roero 2003; Luciano and Roero 2005.

Preface

The Italian mathematician Mario Pieri (1860–1913) played a central role in the research groups of Corrado Segre and Giuseppe Peano, and thus a major role in the development of algebraic geometry and foundations of mathematics in the years around the turn of the twentieth century.

In algebraic geometry, Pieri emphasized birational and enumerative geometry. The thread connecting much of his research is multidimensional projective geometry. Using birational transformations he investigated singular points of algebraic curves and surfaces, classified ruled surfaces, and explored their connections to higher-dimensional projective spaces in innovative ways. His research in enumerative geometry extended that of Hermann Schubert and others. Several of his results are now seen as precursors of modern intersection theory. Although the mainstream of algebraic geometry research departed markedly from Pieri's approach after about 1920, his results nevertheless continue today to stimulate new developments in real enumerative geometry and control-systems theory.

In foundations of mathematics, Pieri created axiomatizations of the arithmetic of natural numbers, of real and complex projective geometry, of inversive geometry, of absolute (neutral) geometry based on the notions of point and motion, and of Euclidean geometry based on point and equidistance. His developments of geometry, born of his critical study of the underpinnings of G. K. C. von Staudt's famous 1847 work *Geometrie der Lage*, differed greatly from other foundations research of Pieri's time. Pieri broke new ground with novel choices of primitive terms. His incisive presentation of his hypothetical-deductive viewpoint, along with contemporary work of others of the Peano school, prepared the scene for deep studies of logical foundations of mathematical theories. The Italians' precise formulations, and the practice of David Hilbert and his followers, established the abstract approach that soon became standard in foundations research and in the exposition of higher mathematics.

Pieri died young, in 1913. There immediately followed thirty-five years of turmoil and catastrophe in Italy and the rest of the world. During that period the work of the Segre and Peano schools lost much of its prominence, and recognition of Pieri's individual contributions disappeared almost entirely from the mathematical literature.

A series of three books is in progress, to examine Pieri's life and mathematical work. This first book, *The Legacy of Mario Pieri in Geometry and Arithmetic*, introduces Pieri and provides an overview of his results. It focuses on his studies in foundations, and provides English translations and analyses of two of his axiomatizations: one in arithmetic and one in geometry. Book two will continue examining Pieri's research in foundations, and will include translations and analyses of two more of his axiomatizations, in absolute and projective geometry. It will provide a thorough analysis of the relationship

of Pieri's philosophy of mathematics to that of other researchers in foundations, in particular Bertrand Russell. The third book will survey the background of Pieri's work in algebraic and differential geometry, placing it in the context of the mathematics of his time. It will then describe his entire research opus in that field, highlighting his influence in such a way that those familiar with more recent work can readily confirm it. Near the end of his life, Pieri entered a different but related field. That final book will also describe his contribution to that effort—to develop Peano's geometrical calculus into a form of vector analysis, with the aim of simplifying much of differential geometry.

The Legacy of Mario Pieri in Geometry and Arithmetic focuses on Pieri's career and his work in foundations, from a viewpoint ninety years after his death. It places his life and research in context and traces his influence on the work of some contemporary and more recent mathematicians. It is addressed to scientists and historians with a general knowledge of logic and advanced mathematics, but with no specialized experience in mathematical logic or foundations of geometry.

Pieri's contributions to foundational studies lie close to the common knowledge of mathematicians today. From this book's concise overviews of Pieri's work and of its relation to that of his contemporaries, readers should have little trouble understanding what Pieri did in this discipline. Moreover, the included translations of his 1907a and 1908a memoirs on the foundations of integer arithmetic and Euclidean geometry will provide an idea of the flavor of Pieri's research and the style of his presentations.

After a first chapter devoted to Pieri's biography and an overview of his entire research career, this book continues in chapter 2 with a survey of his works on foundations of geometry. Chapter 3 contains a complete translation of Pieri's 1908a memoir, *Elementary Geometry Based on the Notions of Point and Sphere*. Next, chapter 4 presents his axiomatization of the arithmetic of natural numbers and its relations to work of Peano and others; section 4.2 is a complete translation of Pieri's 1907a paper *On the Axioms of Arithmetic*.

Chapter 5 evaluates Pieri's impact on some of his contemporaries and on later mathematicians in the foundations area. It explores how his results have been overshadowed, in particular by the research of Peano and Alfred Tarski, even though those two famous mathematicians held Pieri in high esteem. Peano was effusive with praise for Pieri's work. He viewed Pieri's 1907a axiomatization of arithmetic as relegating his own famous 1889 postulates to mere "historical value." Moreover, Peano believed that Pieri's axiomatizations of geometry constituted "an epoch" in such studies that would be a valuable resource for all who followed. Tarski adapted Pieri's approach in the *Point and Sphere* memoir about twenty years later to fit into the elementary logic framework that had crystallized during the 1920s. The turmoil of the times delayed publication of Tarski's system until 1959. Highly successful, Tarski's paper led to a large number of later works in the area.

The authors believe that Pieri's work is not as well known as it should be. Some important histories of mathematics fail to mention his work. Often, when recognized,

it is not discussed in any detail. Often, Pieri is lost among the Italians, with his work attributed merely to the Peano school, or to mathematicians working with Segre. In algebraic geometry, his formulas are well known, but the papers where they appeared are generally not cited, nor is their geometric motivation acknowledged. Pieri's research in foundations has been eclipsed especially by that of Hilbert and his followers, even though Pieri explored some aspects of their work earlier and played a major role in formulating the abstract approach they followed. Finally, the work of various Italian algebraic geometers, including Pieri, has been criticized for generally lacking rigor. It is appropriate now for our project to counter that neglect, and that judgment, by describing and displaying Pieri's work in detail.

The reasons for Pieri's obscurity are complex, and we are careful in our attempts to reconstruct history to explain them. Historical events affected the reception of Pieri's work. The world wars, totalitarianism, and economic depression probably detracted from its dissemination and general recognition. Particular examples are the death, on the first day of World War I, of Louis Couturat, one of Pieri's major French proponents; and the delay of publication of Tarski's [1957] 1959 paper *What Is Elementary Geometry?*, which acknowledged Pieri's contribution, until a decade after the second war. One can also look to Pieri himself for reasons that his work is little known today. This book explores the negative impact of his close association with Peano, his use of Peano's symbolic notation, and of the complexity of Pieri's axiomatizations.

This book's final chapter lists Pieri's published works, lecture notes, and his surviving letters. Detailed annotations or translations describe his reviews, letters, and collected works. The remaining works will be annotated similarly in later books of this series.

There is no organized *Nachlass* of Mario Pieri. Arrighi 1997 is a collection of transcriptions of 132 letters *to* Pieri. The present authors have not been able to locate the originals of those letters. In sections 1.1, 6.6, and 6.7 we have described virtually everything we have found so far in possession of Pieri's relatives and in archives of institutions and of other mathematicians. Section 6.6 describes all thirty-five of his surviving letters. For thirteen of them, and for one review in section 6.7, we have provided complete translations: these constitute their first publication. Material that we find after the present book is published will be reported in the later books of this series.

Style and organizational details. When necessary for clarity or precision, but sparingly, the narrative text in this book employs the concise mathematical terminology and symbolic notation now common in undergraduate courses in higher algebra. The translations in chapter 3 and section 4.2, however, adhere as closely as possible to Pieri's original text; any deviations are noted in those sections. Throughout the book, parenthetical information relating to several sentences in a paragraph may be gathered into a single footnote. The huge bibliography lists all works referred to in the book. Each entry indicates where references occur. The author–date system is employed for citations: for example, “Pieri 1907a” is a citation for a paper Pieri published in 1907. (The present book mentions more than one author named Pieri; citations that include this surname only are references to Mario Pieri.) Sometimes the author is to be inferred from the context,

so that a date alone may also serve as a citation of a work. Biographical sketches of ninety-six individuals closely involved with Pieri and his legacy as described in this book are provided in section 1.3. The book's index lists both subjects and persons. The latter entries include personal dates when known.

Evolution of the project, and acknowledgments. This project to study and explain the life and legacy of Mario Pieri stemmed from Elena A. Marchisotto's 1990 New York University doctoral research, and her correspondence with Francisco Rodriguez-Consuegra. The latter had become interested in Pieri through his 1988 University of Barcelona doctoral research and his 1991 book on Russell. They published a plan for the project in their joint 1993 paper. Over the intervening years, much research and many conversations—particularly with H. S. M. Coxeter, Steven R. Givant, Ivor Grattan-Guinness, Jeremy Gray, Steven L. Kleiman, David E. Rowe, James T. Smith, and Janusz Tarski—led to considerable expansion of the scope of the project. To make this complexity manageable, it was split. Its first part is this book. Subsequent books are in progress, one to consider Pieri's philosophy of mathematics and further aspects of his work on foundations of geometry, and one to survey all his algebraic geometry work in its historical setting. Rodriguez-Consuegra will be a coauthor of the middle book, which is the kernel of the original project. J. T. Smith's contribution to the present book reflects not only his collaboration with Marchisotto on its content, but his assistance with the overall execution of the plan, notably verification of all cited results and publication details, editing, and formatting. Smith is responsible for the annotated translation of Pieri 1908a in chapter 3. For the origin of his interest in Pieri, see a footnote in section 5.2. The authors particularly wish to acknowledge inspiration, assistance, or support by not only the mathematicians and historians of mathematics just mentioned, but also Marco Borga; Francesco, Marco, and Vittorio Campetti; Maria Grazia Ciampini; Salvatore Coen; Helen Cullura Corie; Mario and Mareno Da Collina; Philip J. Davis; Angelo Fabbi; Livia Giacardi; Haragauri N. Gupta; Reuben Hersh; Hubert G. Kennedy; Ann Kostant; Anneli and Peter D. Lax; Joseph A. Marchisotto, Jr.; Pier Daniele Napolitani; Franco Palladino; Patricia Cowan Pearson; Donald Potts; Raffaello Romagnoli; Thomas Sinsheimer; Helen M. Smith; Frank Sottile; Francesco Speranza; Alfred Tarski; and Michael-Markus Toepell. Finally, J. T. Smith is grateful to the Mathematics Department of the University of California at Berkeley for his appointment as visiting scholar, and both authors acknowledge the splendid interlibrary loan service of the California State University.

*Elena A. Marchisotto
James T. Smith
March 2007*

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HISTORY. After the Napoleonic wars, the European powers partially restored to Italy the status quo ante. Austria dominated its northeast, either by direct rule or through states headed by members of the Habsburg (*Asburgo*) family. In the northwest, the Savoy (*Savoia*) family provided kings for Sardinia, a buffer state between Austria and France. It consisted of that island and the Piedmont region of the mainland, with its capital at Turin. The Papal States in the middle of Italy, including Rome, were controlled by the Roman Catholic Church and numerous local dynasties. Austria was also the major influence in the south, the Bourbon (*Borbone*) Kingdom of the Two Sicilies—the island and the southern peninsula—with its capital at Naples. Here and there were enclaves whose political connections and history defied simple classification. LUCCA was one of those.

Napoleon’s conquerors grafted onto Italian roots new social and political institutions. Many persisted through Pieri’s time, and some are still discernible. During the half-century after Napoleon, alliances, invasions, revolutions, industrialization, and social upheaval led unsteadily but inevitably to Italian unification and democratization.

Before Napoleon, Lucca had long been an independent city-state. Afterward, the European powers awarded it to Maria Luisa, widow of the former Bourbon duke of Parma, on condition that her heir be his son, Carlo Lodovico. Parma, which had been ruled by Bourbons before Napoleon, was assigned to the Habsburg princess Maria Luigia, also known as Marie Louise, Napoleon’s second wife. Carlo Lodovico was designated her heir also. Maria Luisa died in 1824. Carlo Lodovico, profligate and despised, was forced out in 1847, just when Maria Luigia died.

Lucca and Parma were then annexed to TUSCANY, the grand duchy of Maria Luigia’s cousin Leopoldo II. (Their paternal grandfather Leopoldo I had been a rather progressive ruler of Tuscany during 1765–1790, and emperor during 1790–1792.) In 1859 a constituent assembly forced Leopoldo II to abdicate. The next year it voted for annexation to Sardinia, and in 1861 the king of Sardinia was proclaimed King Vittorio Emmanuele II of a (partially) unified Italy. For more information on Parma, see page xli.

GEOGRAPHY. Today Italy is divided into twenty *regions*, listed here with their capitals:

Abruzzo	L’Aquila
Apulia (Puglia)	Bari
Basilicata	Potenza
Calabria	Catanzaro
Campania	Naples (Napoli)
Emilia–Romagna	Bologna
Friuli–Venezia Giulia	Trieste
Latium (Lazio)	Rome (Roma)
Liguria	Genoa (Genova)
Lombardy (Lombardia)	Milan (Milano)
Molise	Campobasso
Piedmont (Piemonte)	Turin (Torino)
Sardinia (Sardegna)	Cagliari
Sicily (Sicilia)	Palermo
The Marches (Le Marche)	Ancona
Trentino–South Tyrol (Trentino–Alto Adige)	Trent (Trento)
Tuscany (Toscana)	Florence (Firenze)
Umbria	Perugia
Valley of Aosta (Valle d’Aosta)	Aosta
Venice (Veneto)	Venice (Venezia)

In turn, each region is composed of *provinces*; there are 103 altogether. For example, Tuscany is the region in central Italy between the Arno and Tiber valleys and the Tyrrhenian Sea. It was the locus of the sophisticated Etruscan culture, which flourished hundreds of years before the Roman. The Latin and Italian names for the region are *Etruria* and *Toscana*. Tuscany has ten provinces:

Arezzo	Grosseto	Lucca	Pisa	Prato
Florence (Firenze)	Leghorn (Livorno)	Massa Carrara	Pistoia	Siena



Some Italian cities

- Public universities in Pieri's time
- Other cities



Italy's regions

Life and Works

Mario Pieri was “a true bridge between the two most prestigious Italian schools of mathematics of [his] epoch: that of logic and that of algebraic geometry.”¹ Yet his works are not as well known to today’s scholars as they should be. Pieri left a legacy of results in algebraic and differential geometry, vector analysis, foundations of mathematics, logic, and philosophy of science that are worth knowing, not just for their historical value, but as well for their mathematical and philosophical import.

This is the first of three books on Pieri’s life and work. It discusses his life, presents an overview of his research in foundations of geometry and arithmetic, and provides English translations and analyses of two of his most important papers in those fields. The second book, *The Legacy of Mario Pieri in Logic and Geometry*, will present more detailed accounts of Pieri’s work in philosophy of science and foundations of Euclidean and projective geometry, and will include English translations of two more of his major papers on foundations of geometry. The third, *The Legacy of Mario Pieri in Differential and Algebraic Geometry*, will place in historical context his results in those areas and in vector analysis, including the famous *Pieri formulas*, and will provide English summaries of all of his papers in those fields.

Pieri’s research was valued by his contemporaries and was widely known in his lifetime, even though most of his papers were published in Italian academic journals. Distribution of these was often limited, but their authors generally circulated reprints privately to the mathematical community. Around 1900, Pieri was elected to membership in the academies of science of his home city Lucca, in Tuscany, and of Catania, in Sicily, where he spent eight years as professor.²

At the turn of the twentieth century Pieri’s recognition extended well beyond Italy. For example, because of his contributions to logic and foundations of geometry he was invited to speak at the August 1900 International Congress of Philosophy, in Paris. Unfortunately, he did not attend. Pieri had recently settled in Catania and perhaps was not yet ready to travel so far. He did submit a paper, Pieri [1900]1901, which was summarized in Paris by Louis Couturat, one of the conference organizers.

¹ Brigaglia and Masotto 1982, 135.

² Rindi [1913] 1919, 437, 439.